

MULTIPLYING AND DIVIDING FRACTIONS

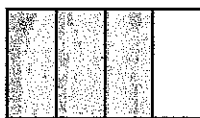
LESSON 1.4

 Multiply and divide fractions.

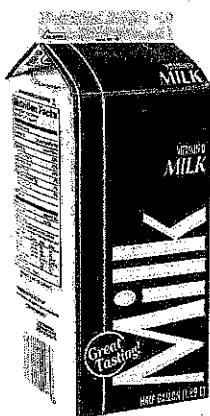
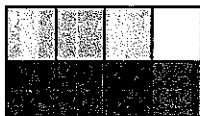
Caroline was making waffles which required $\frac{3}{4}$ cup milk. She wanted to divide the recipe in half because she was only making waffles for two people. How could she find $\frac{1}{2}$ of $\frac{3}{4}$?

In math, the word “of” often means multiply! In this case, Caroline can find $\frac{1}{2}$ of $\frac{3}{4}$ by multiplying $\frac{1}{2} \cdot \frac{3}{4}$. To see a model of this, use a fraction bar:

The shaded part represents $\frac{3}{4}$



To find $\frac{1}{2}$ of $\frac{3}{4}$, divide the rectangle horizontally in half. Shade one of the two horizontal regions with a different color to represent $\frac{1}{2}$.



Count the number of sections that have been shaded twice (where yellow and blue overlap to make green). Three sections out of eight are shaded green. This means that $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{8}$. Caroline will need $\frac{3}{8}$ of a cup of milk to make half the recipe of waffles.

According to the fraction model above, $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$. When you find the product of the numerators, you will have the numerator of the answer. When you multiply the denominators together, you will have the denominator of the answer. This is true for all multiplication problems involving fractions. Some fractions will need to be simplified after multiplying the numerator and denominator.

MULTIPLYING FRACTIONS

For any numbers a , b , c and d :

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

EXAMPLE 1

Find the value of $\frac{2}{3} \cdot \frac{3}{5}$ using models.

SOLUTION

Draw a model and shade $\frac{3}{5}$ of a rectangle. Draw horizontal lines to cut the model into thirds. Shade 2 of the three horizontal rectangles to represent $\frac{2}{3}$.



The final model has 6 out of 15 regions which are shaded twice, so $\frac{2}{3} \cdot \frac{3}{5} = \frac{6}{15}$.

In simplest form, $\frac{6}{15} = \frac{2}{5}$.

EXAMPLE 2

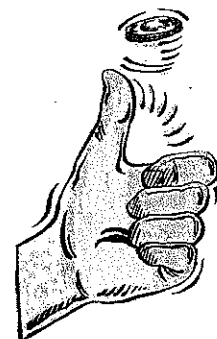
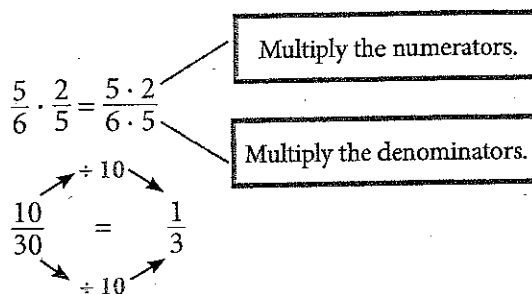
Find the value of $\frac{5}{6} \cdot \frac{2}{3}$.

SOLUTION

Multiply.

Simplify.

$$\frac{5}{6} \cdot \frac{2}{3} = \frac{1}{3}$$



Two numbers are **reciprocals** if their product is 1. To find the reciprocal of a fraction, “flip” the fraction so that the numerator becomes the denominator and the denominator becomes the numerator.

$$\frac{3}{4} \xrightarrow{\text{Reciprocal}} \frac{4}{3}$$

$$\frac{1}{2} \xrightarrow{\text{Reciprocal}} \frac{2}{1}$$

Reciprocals are used when dividing fractions. To divide by a fraction, multiply by its reciprocal.

DIVIDING FRACTIONS

For any numbers a, b, c and d :

$$\frac{a}{b} \div \frac{c}{d} = \frac{a \cdot d}{b \cdot c}$$

EXAMPLE 3Find the value of $\frac{3}{5} \div \frac{7}{10}$.**SOLUTION**

Divide.

$$\frac{3}{5} \div \frac{7}{10} = \frac{3}{5} \cdot \frac{10}{7} = \frac{30}{35}$$

Simplify.

$$\frac{30}{35} = \frac{6}{7}$$

$$\frac{3}{5} \div \frac{7}{10} = \frac{6}{7}$$

EXAMPLE 4Find the value of $\frac{5}{6} \div \frac{1}{8}$.**SOLUTION**

Divide.

$$\frac{5}{6} \div \frac{1}{8} = \frac{5}{6} \cdot \frac{8}{1} = \frac{40}{6}$$

Simplify.

$$\frac{40}{6} = \frac{20}{3}$$

Change into a mixed number.

$$\begin{array}{r} 6 \\ 3 \overline{)20} \\ \underline{-18} \\ 2 \end{array}$$

$$\frac{5}{6} \div \frac{1}{8} = 6\frac{2}{3}$$