## Connections

## T3xtensions

## Applications

1. At some international airports, trains carry passengers between the separate terminal buildings. Suppose that one such train system moves along a track like the one below.

a. A train leaves the main terminal going east at 10 meters per second. Where will it be in 10 seconds? When will it reach the east terminal?
b. A train passes the main terminal going east at 10 meters per second. Where was that train 15 seconds ago? When was it at the west terminal?
c. A train leaves the main terminal going west at 10 meters per second. Where will it be in 20 seconds? When will it reach the west terminal?
d. A train passes the main terminal going west at 10 meters per second. When was it at the east terminal? Where was it 20 seconds ago?


The dot patterns illustrate commutative properties for operations on whole numbers. Write a number sentence for each case.
2.

3.

4. Find each value.
a. $7 \cdot 2$
b. $-7 \times(-2)$
c. $7 \times(-2)$
d. $-7 \times 2$
e. $8 \cdot 2.5$
f. $-9 \times(-4)$
g. $12 \times(-3)$
h. $-1.5 \times 4$
i. $3.5 \times 7$
j. $-8.1 \cdot(-1)$
k. $1 \times(-6)$
I. $-2 \frac{1}{2} \times 1$
5. Find the values for each pair.
a. $4 \times(-3)$ and $-3 \times 4$
b. $2 \cdot(-4)$ and $-4 \cdot 2$
c. $-2 \times(-3)$ and $-3 \times(-2)$
d. $\frac{1}{5} \times\left(-\frac{4}{9}\right)$ and $-\frac{4}{9} \times \frac{1}{5}$
e. What can you conclude about multiplication with negative numbers?
6. Tell whether each product is greater than or less than zero.
a. $5 \times(-7)$
b. $-3.2 \cdot 1.5$
c. $10.5 \times(-4)$
d. $-2 \times(-3) \times(-1)$
e. $-\frac{2}{3} \cdot 2 \frac{3}{4}$
f. $-\frac{3}{4} \times\left(-1 \frac{5}{6}\right) \times\left(-\frac{7}{4}\right)$
g. $-\frac{3}{4} \times\left(-1 \frac{5}{6}\right) \times \frac{7}{4}$
h. $-\frac{3}{4} \times\left(-1 \frac{5}{6}\right) \times\left(-\frac{7}{4}\right) \times\left(-2 \frac{3}{8}\right)$
i. $\frac{3}{4} \cdot\left(-1 \frac{5}{6}\right) \cdot \frac{7}{4} \cdot\left(-2 \frac{3}{8}\right)$
j. $\frac{3}{4} \times 1 \frac{5}{6} \times \frac{7}{4} \times\left(-2 \frac{3}{8}\right)$
7. You have located fractions such as $-\frac{5}{7}$ on a number line. You have also used fractions to show division: $\frac{-5}{7}=-5 \div 7$ and $\frac{5}{-7}=5 \div(-7)$. Tell whether each statement is true or false. Explain.
a. $\frac{-1}{2}=\frac{1}{-2}$
b. $-\frac{1}{2}=\frac{-1}{-2}$
8. Find a value for $n$ to make each sentence true.
a. $24 \div 2=n$
b. $-24 \div(-2)=n$
c. $24 \div n=-12$
d. $n \div 2=-12$
e. $5 \div 2.5=n$
f. $-12 \div n=3$
g. $n \div(-3)=-4$
h. $-16 \div \frac{1}{4}=n$

Write four related multiplication and division facts for each set of integers.
Sample 27, 9, 3

$$
\begin{aligned}
9 \times 3 & =27 \\
3 \times 9 & =27 \\
27 \div 9 & =3 \\
27 \div 3 & =9
\end{aligned}
$$

9. $7,-3,-21$
10. $-4,-5,20$
11. $1.5,-3,-4.5$

Without doing any calculations, determine whether each expression is greater than, less than, or equal to 0 .
12. $-1,105.62 \div 24.3$
13. $0 \times(-67)$
14. $-27.5 \times(-63)$
15. $0 \div 89$
16. $-54.9 \div(-3)$
17. $-2,943 \times 1.06$
18. Use the algorithms you developed to find each value. Show your work.
a. $12 \cdot 9$
b. $5 \times(-25)$
c. $-220 \div(-50)$
d. $48 \div(-6)$
e. $-63 \div 9$
f. $\frac{2}{-3} \times\left(-\frac{4}{5}\right)$
g. $\frac{-99}{33}$
h. $-2.7 \div(-0.3)$
i. $-36 \times 5$
j. $52.5 \div(-7)$
k. $-2 \frac{1}{2} \times\left(-\frac{2}{3}\right)$
I. $9 \div 5$
m. $-9 \times(-50)$
n. $-\frac{96}{24}$
0. $6 \times 1 \frac{1}{2}$
p. $-\frac{5}{8} \times \frac{8}{5}$
q. $4 \times\left(-1 \frac{1}{4}\right)$
r. $-2.5 \times 2 \frac{1}{5}$

## Go Online <br> PHSchool.com

For: Multiple-Choice Skills Practice
Web Code: ana-4354

## Multiple Choice Find each value.

19. $-24 \div 4$
A. -96
B. -6
C. 6
D. 96
20. $-10 \times(-5)$
F. -50
G. -2
H. 2
J. 50
21. Chris and Elizabeth are making a version of the Integer Product Game in which players need three products in a row to win. What six factors do they need for their game?

Chris and Elizabeth's Product Game

| 4 | -4 | 6 | -6 |
| :---: | :---: | :---: | :---: |
| 9 | -9 | 10 | -10 |
| 15 | -15 | 25 | -25 |

For: Help with Exercise 21 Web Code: ane-4321

## Connections

22. Multiply or divide. Show your work.
a. $52 \times 75$
b. $52 \times(-75)$
c. $-2,262 \div(-58)$
d. $\frac{2}{3} \times \frac{4}{5}$
e. $-9,908 \div 89$
f. $-7.77 \div(-0.37)$
g. $-34 \times 15$
h. $53.2 \div(-7)$
i. $-\frac{2}{3} \times \frac{6}{8}$
j. $90 \div 50$
k. $-90 \times(-50)$
I. $-108 \div 24$
m. $19.5 \div(-3)$
n. $-8.4 \times 6$
O. $6 \times 2 \frac{1}{2}$
p. $-3 \frac{2}{3} \times(-9)$
q. $-4 \times\left(1 \frac{1}{4}\right)$
r. $-2.5 \times-2 \frac{1}{5}$
23. Find integers to make each sentence true.
a. $\times \square=30$
b. $\square \times \square=-30$
c. $-24 \div \square=\square$
24. On Tuesday, the temperature changes $-2^{\circ} \mathrm{F}$ per hour from noon until 10:00 a.m. the next morning. The temperature at noon on Tuesday is $75^{\circ} \mathrm{F}$.
a. What is the temperature at 4:00 p.m. on Tuesday?
b. What is the temperature at 9:00 a.m. on Wednesday?
c. Plot the (time, temperature) data on a coordinate graph using noon Tuesday as time 0 .
d. Describe the pattern of points. How does the pattern relate to the rate of change in temperature?

25. The diagram below shows Mug Wump drawn on a coordinate grid.

a. Complete the $(x, y)$ column of a table like the one shown to record coordinates of key points needed to draw Mug, or copy your table from Exercise 34 of Investigation 2.

Coordinates for Mug and Variations

| Rule | $(x, y)$ | $(2 x, 2 y)$ | $(-2 x,-2 y)$ |
| :---: | :---: | :---: | :---: |
| Head <br> Outline | $(-4,-2)$ |  |  |
|  | $(-2,-2)$ |  |  |
|  | $(-2,-3)$ |  |  |
|  |  |  |  |
| Nose | $(-1,1)$ |  |  |
|  |  |  |  |
| Mouth | $(-2,-1)$ |  |  |
|  |  |  |  |
|  |  |  |  |
| Eyes | $(-2,2)$ |  |  |
|  |  |  |  |

b. Suppose you make scale drawings with rules $(x, y) \rightarrow(2 x, 2 y)$ and $(x, y) \rightarrow(-2 x,-2 y)$. Give coordinates for the images of Mug.
c. On graph paper, plot the images of Mug Wump produced by the new sets of coordinates in part (b).
d. Compare the length, width, and area of Mug's mouth to those of the figures drawn in part (c). Explain how you could have predicted those results by studying the coordinate rules for the drawings.
26. Write a number sentence to represent each situation.
a. The Extraterrestrials have a score of -300 . They answer four 50-point questions incorrectly. What is their new score?
b. The Super Computers answer three 100-point questions incorrectly. They now have 200 points. What was their score before answering the three questions?
c. The Bigtown Bears football team are at the 25 -yard line. In the next three plays, they lose an average of 4 yards per play. Where are the Bears after the three plays?
d. A new convenience store wants to attract customers. For a one-day special, they sell gasoline for $\$ 0.25$ below their cost. They sell 5,750 gallons that day. How much money do they lose?

27. The list below gives average temperatures (in ${ }^{\circ} \mathrm{C}$ ) for Fairbanks, Alaska, for each month of the year, from January through December.
$-25,-20,-13,-2,9,15,17,14,7,-4,-16,-23$
a. What is the median?
b. What is the range?
c. What is the mean?
d. Number the months from 1 (for January) through 12 (for December). Plot a graph of the (month, temperature) data.
28. Find the sum, difference, product, or quotient without using a calculator.
a. $-5-18$
b. $-23+48$
c. $\frac{3}{4} \times\left(\frac{-5}{9}\right)$
d. $119+(-19.3)$
e. $-1.5-(-32.8)$
f. $12 \div 15$
g. $-169 \div(-1.3)$
h. $0.47-1.56$
i. $6 \times(-3.5)$
j. $\frac{2}{-3} \div \frac{5}{6}$
k. $\frac{7}{12}-\left(-\frac{2}{3}\right)$
I. $-\frac{4}{5}+\left(-\frac{1}{4}\right)$
29. Estimate the sum, difference, product, or quotient.
a. $-52-5$
b. $-43+(-108)$
c. $2 \frac{3}{4} \times\left(-\frac{5}{9}\right)$
d. $79+(-25.3)$
e. $-12.5-(-37.3)$
f. $89 \div 15$
g. $-169 \div(-13)$
h. $6.3-1.86$
i. $61 \times(-3.9)$
j. $-\frac{2}{3} \div 1 \frac{5}{6}$
k. $5 \frac{7}{12}-\left(-\frac{2}{3}\right)$
I. $-\frac{4}{5} \div\left(-\frac{1}{4}\right)$

## Extensions

30. Many towns and small cities have water towers to store water. Water flows into and out of the towers all day long. Generally, flow out of the tower is greatest during the hours when most people are awake and active. The flow into the tower is greatest at night when most people are asleep.

The table below shows the water flow into and out of a water tower for a given time period. For each part, write a number sentence to find the change in water supply over the given time.

Water Tower Water Flow

|  | Water Flow In <br> (gallons per hour) | Water Flow Out <br> (gallons per hour) | Time <br> (hours) |
| :--- | :---: | :---: | :---: |
| a. | 5,000 | 0 | 4 |
| b. | 4,000 | 0 | 7 |
| c. | 0 | 7,500 | 3 |
| d. | 5,000 | 3,000 | 6.5 |
|  |  |  |  |


31. To add $5+3+2$, you might think that it is easier to add the $3+2$ and then add the answer to the 5 . The mathematical property that allows you to change the grouping of addends (or factors) is called the Associative Property.

Test the Associative Property for addition and multiplication of integers by simplifying below. Find the values within the parentheses first. When you need a grouping symbol like parentheses inside another parentheses, you can use brackets to make it easier to read. For example, $(4-(-6))$ can be written as $[4-(-6)]$.
a. $[3 \times(-3)] \times 4$ and $3 \times(-3 \times 4)$
b. $(-5 \times 4) \times(-3)$ and $-5 \times[4 \times(-3)]$
c. $[-2 \times(-3)] \times(-5)$ and $-2 \times[-3 \times(-5)]$
d. $(3 \times 4) \times(-5)$ and $3 \times[4 \times(-5)]$
e. $[3+(-3)]+4$ and $3+(-3+4)$
f. $(-5+4)+(-3)$ and $-5+[4+(-3)]$
g. $[-2+(-3)]+(-5)$ and $-2+[-3+(-5)]$
h. $(3+4)+(-5)$ and $3+[4+(-5)]$
i. Does the Associative Property work for addition and multiplication of integers?
32. Explain how each rule changes the original shape, size, and location of Mug Wump.
a. $(x, y) \rightarrow(-x, y)$
b. $(x, y) \rightarrow(x,-y)$
c. $(x, y) \rightarrow(-0.5 x,-0.5 y)$
d. $(x, y) \rightarrow(-0.5 x, y)$
e. $(x, y) \rightarrow(-3 x,-3 y)$
f. $(x, y) \rightarrow(-3 x+5,-3 y-4)$

33. Tell whether each statement is true or false. Explain.
a. $-1=-1+0$
b. $-3 \frac{3}{8}=-\frac{21}{8}$
c. $-6.75=-6+\left(-\frac{3}{4}\right)$
34. Find a set of numbers to make a Sum Game. Each sum on the board should be the sum of two numbers (possibly a single number added to itself). Each pair of numbers should add to a sum on the board.
Hint: You need 11 numbers, all with different absolute values.
Sum Game Board

| -24 | -22 | -20 | -18 | -16 | -14 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -12 | -11 | -10 | -9 | -8 | -7 |
| -6 | -5 | -4 | -3 | -2 | -1 |
| 0 | 1 | 2 | 3 | 4 | 5 |
| 6 | 7 | 8 | 9 | 10 | 11 |
| 12 | 14 | 16 | 18 | 20 | 22 |

## Numbers:

35. Write a story for a problem that is answered by finding the value of $n$.
a. $-4 n=-24$
b. $\frac{n}{2}=16$
