Investigation 4

Properties of Operations

When you learn new types of numbers, you want to know what properties apply to them. You know that rational numbers are commutative for addition and multiplication.

$$-\frac{2}{3}+\frac{1}{6}=\frac{1}{6}+\left(-\frac{2}{3}\right) \text{ and } -\frac{2}{3}\times\frac{1}{6}=\frac{1}{6}\times\left(-\frac{2}{3}\right)$$

In this investigation, you will study another important property of rational numbers. You will also learn a mathematical rule that tells you the order in which to do arithmetic operations.

4.1 Order of Operations

Mathematicians have established rules called the **order of operations** in which to perform operations ($+, -, \times, \div$). Why do you need such rules?

Getting Ready for Problem 4.1

The rugby club orders 20 new jerseys. The manufacturer charges a $100 setup fee and $15 per shirt. The total cost is represented by the equation, $C = 100 + 15n$, where C is the cost in dollars and n is the number of jerseys ordered. Pedro and David calculate the amount the club owes.

Pedro's calculation:
$$\begin{aligned} C &= 100 + 15 \times 20 \\ &= 100 + 300 \\ &= \$400 \end{aligned}$$

David's calculation:
$$\begin{aligned} C &= 100 + 15 \times 20 \\ &= 115 \times 20 \\ &= \$2{,}300 \end{aligned}$$

- Who did the calculations correctly?

Order of Operations

1. Compute any expressions within parentheses.

Example 1

$(-7 - 2) + 1 =$

$-9 + 1 = -8$

Example 2

$(1 + 2) \times (-4) =$

$3 \times (-4) = -12$

2. Compute any exponents.

Example 1

$-2 + 3^2 =$

$-2 + 9 = 7$

Example 2

$6 - (-1 + 4)^2 =$

$6 - (3)^2 = -3$

3. Multiply and divide in order from left to right.

Example 1

$1 + 2 \times 4 =$ Multiplication first

$1 + 8 = 9$

Example 2

$200 \div 10 \times 2 =$ Division first

$20 \times 2 = 40$ Multiplication second

4. Add and subtract in order from left to right.

$1 + 2 - 3 \times 4 =$ Multiplication first

$1 + 2 - 12 =$ Addition and subtraction

$3 - 12 = -9$

Use the order of operations in Problem 4.1.

Problem 4.1 Order of Operations

A. In a game, the goal is to write a number sentence that gives the greatest possible result using all the numbers on four cards. Jeremy draws the following four cards.

1. Joshua writes $5 - (-6) \times 4 + (-3) = 41$. Sarah says the result should be 26. Who is correct and why?
2. Wendy starts by writing $-3 - (-6) + 5^4 =$. What is her result?
3. Insert parentheses into $-3 - (-6) + 5^4$ to give a greater result than in part (2).

B. Find each value.

1. $-7 \times 4 + 8 \div 2$
2. $(3 + 2)^2 \times 6 - 1$
3. $2\frac{2}{5} \times 4\frac{1}{2} - 5^3 + 3$
4. $8 \times (4 - 5)^3 + 3$
5. $-8 \times [4 - (-5 + 3)]$
6. $-16 \div 8 \times 2^3 + (-7)$

C. Use parentheses, if needed, to make the greatest and least possible values.

1. $7 - 2 + 3^2$
2. $46 + 2.8 \times 7 - 2$
3. $25 \times (-3.12) + 21.3 \div 3$
4. $5.67 + 35.4 - 178 - 181$

D. Use the order of operations to solve this problem. Show your work.

$$3 + 4 \times 5 \div 2 \times 3 - 7^2 + 6 \div 3 = \blacksquare$$

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4.2 Distributing Operations

In this problem, you will compute areas of rectangles using different expressions. Look for ways to rewrite an expression into an equivalent expression that is easier to compute.

Problem 4.2 Distributing Operations

A. Richard lives in a neighborhood with a rectangular field. Each part below shows a way to divide the field for different kinds of sports.

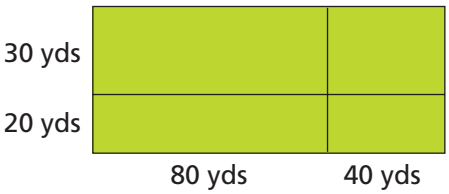
1. Find the area.

2. The field is divided into two parts.

a. Find the area of each part.

b. Write a number sentence that shows that the sum of the smaller areas is equal to the area of the entire field.

3. The field is divided into four parts.

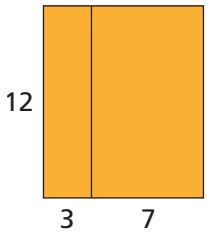


a. Find the area of each part.

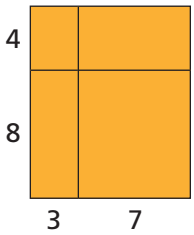
b. Write a number sentence that shows that the sum of the smaller areas is equal to the area of the entire field.

B. Use what you learned in Question A. Write two different expressions to find the area of each rectangle. Tell which uses fewer operations.

1.



2.



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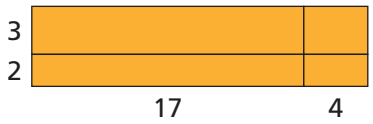

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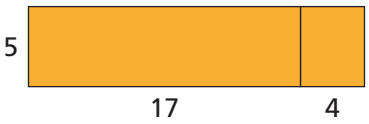


C. **1.** Draw a rectangle whose area can be represented by $7 \times (11 + 9)$.

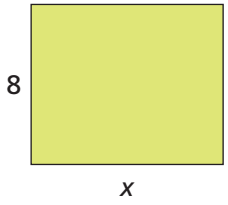
2. Write another expression for the area of the rectangle in part (1).

3. Draw a rectangle whose area can be represented by $(3 + 1) \times (3 + 4)$.

4. Write another expression for the area of the rectangle in part (3).

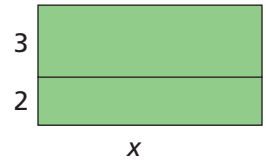
D. The unknown length in each rectangle is represented by a variable x.

1. Write an expression to represent the area of the rectangle.



2. Write two different expressions to represent the area of each rectangle below.

a.



b.

E. Find the missing part(s) to make each sentence true.

1. $12 \times (6 + 4) = (12 \times \square) + (12 \times 4)$

2. $2 \times (n + 4) = (2 \times \square) + (\square \times 4)$

3. $(n \times 5) + (n \times 3) = \square \cdot (5 + 3)$

4. $(-3 \times 5) + (\square \times 7) = -3 \cdot (\square + 7)$

5. $4n + 11n = n \cdot (\square + \square)$

ACE **Homework starts on page 69.**

4.3 The Distributive Property and Subtraction

The rectangles in Problem 4.2 illustrate an important property of numbers and operations called the **Distributive Property.** This property shows that multiplication *distributes* over addition. When you think about a multiplication problem like 512×5 as $500 \times 5 + 12 \times 5$, or $12 \times 5\frac{3}{4}$ as $12 \times 5 + 12 \times \frac{3}{4}$, you are using the Distributive Property.

Getting Ready for Problem 4.3

You can use the Distributive Property to rewrite an expression as one that is easier to calculate or gives new information. You can do this in two ways.

1. Suppose an expression is written as the product of two factors, one of which is a sum. You can use the Distributive Property to multiply one factor by each number in the second factor. This is called *expanding* the expression.

   $$-3 \cdot (4 + 8) = -3 \cdot 4 + (-3) \cdot 8$$

   With a variable: $-2 \cdot (x + 6) = -2x + (-2) \cdot 6$

2. Suppose an expression is written as a sum and the numbers have a common factor. You can use the Distributive Property to rewrite the expression as the common factor multiplied by the sum. This is called *factoring* the expression.

   $$5 \cdot 4 + 5 \cdot 7 = 5 \cdot (4 + 7)$$

   With a variable: $8 \cdot 2 + 8x = 8 \cdot (2 + x)$

- Do you think the Distributive Property can be used to expand or factor expressions with subtraction? Explain your reasoning.

Problem 4.3 The Distributive Property and Subtraction

A. Use the Distributive Property to expand each expression.

1. $5 \cdot (3 + 2)$

2. $5 \cdot [3 + (-2)]$

3. $5 \cdot (3 - 2)$

4. $5 \cdot [3 - (-2)]$

5. For parts (1)–(4), find the value of the expression.

6. Does the Distributive Property seem to hold for subtraction? Explain.

B. Use the Distributive Property to expand each expression.

1. $-5 \cdot (3 + 2)$

2. $-5 \cdot (3 - 2)$

3. $-5 \cdot [3 + (-2)]$

4. $-5 \cdot [3 - (-2)]$

5. For parts (1)–(4), find the value of the expression.

6. Explain how to distribute a negative number to expand an expression.

C. Write each expression in factored form.

1. $6 \cdot 2 + 6 \cdot 3$

2. $6 \cdot 2 - 6 \cdot 3$

3. $-6 \cdot 2 + (-6) \cdot 3$

4. $-6 \cdot 2 - (-6) \cdot 3$

5. $5x - 8x$

6. $-3x - 4x$

7. Explain how to factor an expression with subtraction.

D. Three friends are going hiking. Lisa buys 2 bottles of water and 3 packs of trail mix for each of them.

1. Can she go through the express checkout lane for customers with 15 or fewer items?

2. Write a number sentence to show how you found the total number of items.

3. Write another number sentence to find the total number of items.

E. Mr. Chan bought a roll of kitchen towels for $1.19 and window cleaner for $2.69. In his state there is a 4% sales tax on these items.

1. What is his total bill?

2. Write a number sentence to show how you found the total bill.

3. Suppose you add the prices of the two items and then compute the tax. Your friend finds the tax on each item and then adds the two together. Which method is better? Explain.

ACE **Homework starts on page 69.**

More on Notation

Now you can use the order of operations or the Distributive Property to find the value of an expression like $-8 \cdot [-2 + (-3)]$ that has parentheses.

Order of operations method:

$$-8 \cdot [-2 + (-3)] = -8 \cdot (-5) \qquad \text{Add } -2 \text{ and } -3 \text{ within the parentheses.}$$

$$= 40 \qquad \text{Multiply.}$$

Distributive Property method:

$$-8 \cdot [-2 + (-3)] = -8 \cdot (-2) + (-8) \cdot (-3) \qquad \text{Expand first.}$$

$$= 16 + 24 \qquad \text{Multiply.}$$

$$= 40$$

Either method is correct.