

Investigation

4

Making Sense of Proportions

In the following comparison problems, you have information about the relationship between quantities, but one or more specific values are unknown.

- **Calculators** Calculators are on sale at a price of \$1,000 for 20. How many can be purchased for \$1,250?
- **Similar Figures** The scale factor relating two similar figures is 2. One side of the larger figure is 10 centimeters long. How long is the corresponding side of the smaller figure?
- **Country Music** Country music is the primary format of 20% of American radio stations. There are about 10,600 radio stations in the United States. About how many stations focus on country music?
- **Doctors** Among American doctors, males outnumber females by a ratio of 15 to 4. If about 450,000 doctors are males, about how many are females?

Each of these problems can be solved in several ways. You will learn specific ways to set up ratios for problems like this and find missing values.



4.1

Setting Up and Solving Proportions

There are many ways to solve problems such as the ones on the previous page. One standard way is to create two ratios to represent the information in the problem. Then set these two ratios equal to each other to form a proportion. A **proportion** is an equation that states two ratios are equal.

For example, in the problem about doctors, you have enough information to write one ratio. Then write a proportion to find the missing quantity. There are four different ways to write a proportion representing the data in the problem.

Write the known ratio of male to female doctors. Complete the proportion with the ratio of actual numbers of doctors.

$$\frac{15 \text{ (male)}}{4 \text{ (female)}} = \frac{450,000 \text{ males}}{x \text{ females}}$$

Write a ratio of male to male data. Complete the proportion with female to female data.

$$\frac{15 \text{ (male)}}{450,000 \text{ males}} = \frac{4 \text{ (female)}}{x \text{ females}}$$

Write the known ratio of female to male doctors. Complete the proportion with the ratio of actual numbers of doctors.

$$\frac{4 \text{ (female)}}{15 \text{ (male)}} = \frac{x \text{ females}}{450,000 \text{ males}}$$

Write a different ratio of male to male data. Complete the proportion with female to female data.

$$\frac{450,000 \text{ males}}{15 \text{ (male)}} = \frac{x \text{ females}}{4 \text{ (female)}}$$

Using your knowledge of equivalent ratios, you can now find the number of female doctors from any one of these proportions.

Does any arrangement seem easier than the others?

Getting Ready for Problem 4.1

Analyze the “Similar Figures” problem in the introduction.

The scale factor relating two similar figures is 2. One side of the larger figure is 10 centimeters long. How long is the corresponding side of the smaller figure?

- The scale factor means that the lengths of the sides of the larger figure are 2 times the lengths of the sides of the smaller. What is the ratio of the side lengths of the smaller figure to those of the larger figure?
- Write a proportion to represent the information in the problem.
- Solve your proportion to find the length of the corresponding side of the smaller figure.

Problem 4.1 Setting Up and Solving Proportions

- A.** Figure out whether each student’s thinking about each line in the following problem is correct. Explain.

Dogs outnumber cats in an area by a ratio of 9 to 8. There are 180 dogs in the area. How many cats are there?

Adrianna’s Work:

$$\frac{9 \text{ dogs}}{8 \text{ cats}} = \frac{180 \text{ dogs}}{x \text{ cats}}$$

$$\frac{9}{8} \times \frac{20}{20} = \frac{180}{160}$$

$$\frac{180}{160} = \frac{180}{x}$$

$$x = 160$$

1. Why did Adrianna multiply by $\frac{20}{20}$? How did she find what to multiply by?
2. What does this proportion tell you about the denominators? Why?
3. Is the answer correct? Explain.

Joey’s Work:

$$\frac{8 \text{ cats}}{9 \text{ dogs}} = \frac{x \text{ cats}}{180 \text{ dogs}}$$

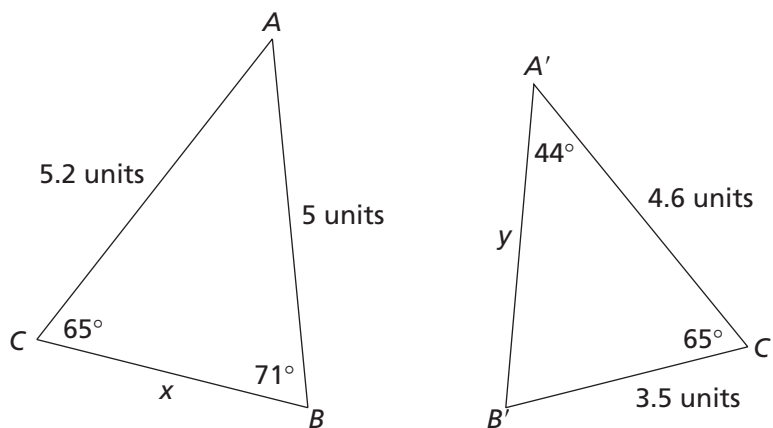
$$\frac{8}{9} = \frac{80}{90} = \frac{160}{180}$$

There are 160 cats.

4. What strategy did Joey use?
5. Why can he make this claim?

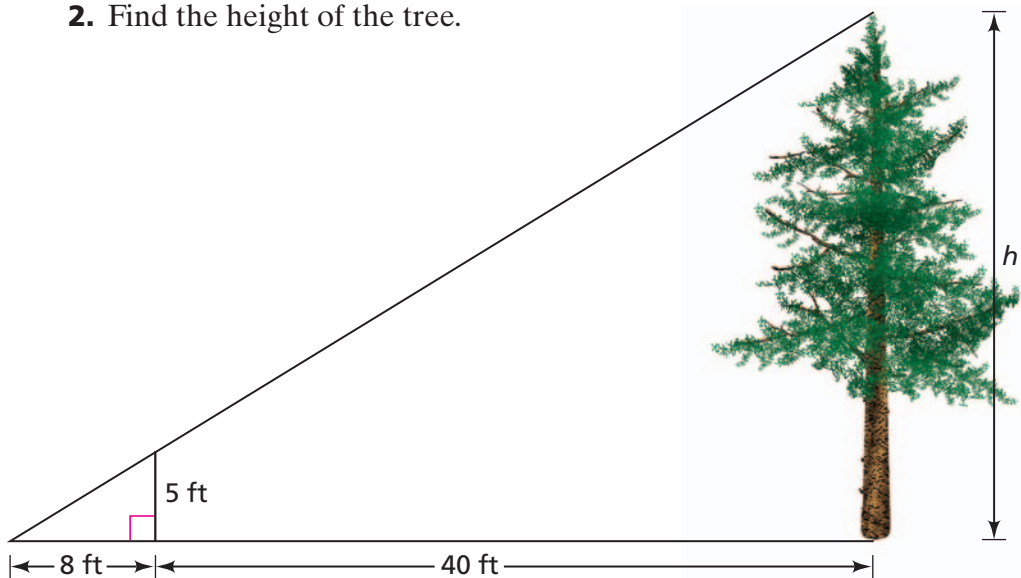
- B. 1.** Calculators are on sale at a price of \$1,000 for 20. How many can be purchased for \$1,250? Write and solve a proportion that represents the problem. Explain.
- 2.** Country music is the primary format of 20% of American radio stations. There are about 10,600 radio stations in the United States. About how many stations focus on country music?
- C.** Use the reasoning you applied in Question B to solve these proportions for the variable x . Explain.
- 1.** $\frac{8}{5} = \frac{32}{x}$ **2.** $\frac{7}{12} = \frac{x}{9}$ **3.** $\frac{25}{x} = \frac{5}{7}$ **4.** $\frac{x}{3} = \frac{8}{9}$
- D.** Use proportions to find the missing lengths in the following similar shapes.

1.



active math
online
For: Scaling Figures Activity
Visit: PHSchool.com
Web Code: and-3401

2. Find the height of the tree.



ACE Homework starts on page 55.

4.2 Everyday Use of Proportions

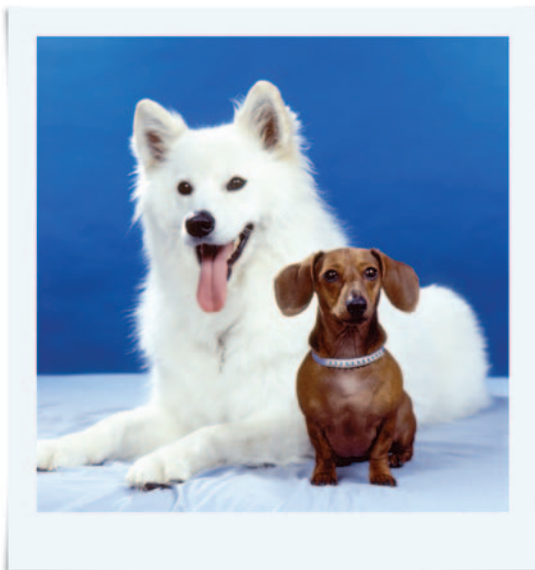
In our everyday lives, we often need to solve proportion problems. So do bakers, tailors, designers, and people in many other occupations.

You may have heard someone say, “A pint is a pound the world around.” This saying suggests how to compare liquid measures with weight. It tells us that a pint of liquid weighs about a pound. If you drink a quart of milk a day, you might ask,

“About how much does a quart of liquid weigh?”

Problem 4.2 Applications of Proportions

- A.** Jogging 5 miles burns about 500 Calories. How many miles will Tanisha need to jog to burn off the 1,200-Calorie lunch she ate?
- B.** Tanisha jogs about 8 miles in 2 hours. How long will it take her to jog 12 miles?
- C.** Sam’s grandmother says that “a stitch in time saves nine.”
 - 1.** What do you think Sam’s grandmother means?
 - 2.** Sam’s grandmother takes 25 stitches in time. How many does she save?
- D.** Imani gives vitamins to her adult dogs. The recommended dosage is 2 teaspoons per day for adult dogs weighing 20 pounds. She needs to give vitamins to Bruiser, who weighs 75 pounds, and to Dust Ball, who weighs 7 pounds. What is the correct dosage for each dog?



- E. The scale factor relating two similar figures is 1.8. One side of the larger figure is 12 centimeters. How long is the corresponding side of the smaller figure?

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4.3 Developing Strategies for Solving Proportions

When mathematicians find the same kind of problem occurring often, they look for a systematic method, or algorithm, that can be applied in each case.

So far in this investigation, you have found ways to solve proportions in specific cases with nice numbers. Now you will develop general strategies that will guide you in solving proportions when the numbers are not so nicely related.

Problem 4.3 Developing Strategies for Solving Proportions

- A.** A jet takes 10 miles to descend 4,000 feet. How many miles does it take for the jet to descend 5,280 feet?
1. Set up two different proportions that can be solved to answer the question.
 2. Solve one of your proportions by whatever method you choose. Check to see that your answer makes sense.
- B.** Jack works at a restaurant and eats one enchilada for lunch every day that he works. He figures that he ate 240 enchiladas last year. Three enchiladas have a total of 705 Calories. How many Calories did he take in last year from eating enchiladas?
1. Set up a proportion that can be solved to answer the question.
 2. Solve your proportion. Check to see that your answer makes sense.
 3. Describe each step in your solution strategy.
 4. Can your strategy be used to solve any proportion? Explain.
 5. How many Calories did he eat for lunch each working day?

- C.** In Pinecrest Middle School, there are 58 sixth-graders, 76 seventh-graders, and 38 eighth-graders. The school council is made up of 35 students who are chosen to represent all three grades fairly.
1. Write fractions to represent the part of the school population that is in each grade.
 2. Use these fractions to write and solve proportions that will help you determine a fair number of students to represent each grade on the school council. Explain.
 3. How would the number of students from each grade change if the number of members of the school council were increased to 37? Explain your reasoning.
- D.** Ms. Spencer needs 150 graphing calculators for her math students. Her budget allows \$5,000 for calculators. She needs to know if she can buy what she needs at the discount store where calculators are on sale at 8 for \$284.

She writes the following statement:

$$\frac{8}{284} = \frac{150}{x} \quad \text{or} \quad \frac{8}{284} = 150 \div x$$

1. Use fact-family relationships to rewrite the proportion so that it is easier to find x .
2. Solve the proportion, recording and explaining each of your steps.
3. Is your method a general method that can be used to solve any proportion? Explain.

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