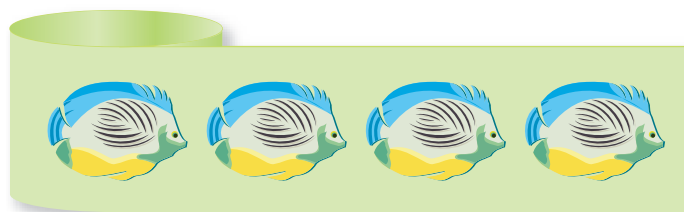
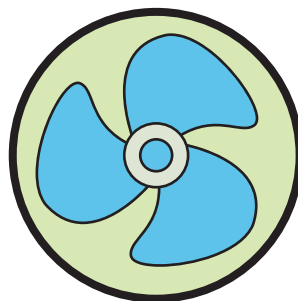
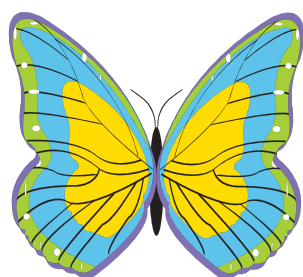


## Three Types of Symmetry

**W**hen part of a design is repeated to make a balanced pattern, we say the design has **symmetry**. Artists use symmetry to make designs that are pleasing to the eye. Architects use symmetry to produce a sense of balance in their buildings. Symmetry is also a feature of animals, plants, and mechanical objects.

The butterfly, fan, and ribbon below illustrate three kinds of symmetry.



### Getting Ready for Problem 1.1

- What part of each design is repeated to make a balanced pattern that allows us to say the three figures have symmetry?
- How do the figures suggest different kinds of symmetry?

## 1.1 Reflection Symmetry

You have probably made simple heart shapes by folding and cutting paper as shown below.



The resulting heart shape has **reflection symmetry**, which is sometimes called *mirror symmetry* or *line symmetry*. The fold shows the **line of symmetry**. A line of symmetry divides a figure into halves that are mirror images.

If you place a mirror on a line of symmetry, you will see half of the figure reflected in the mirror. The combination of the half-figure and its reflection will have the same size and shape as the original figure. You can use a mirror to check a design for symmetry and to locate the line of symmetry.

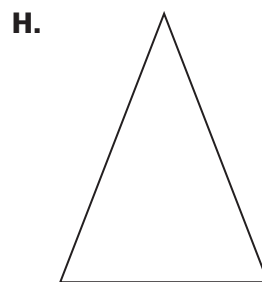
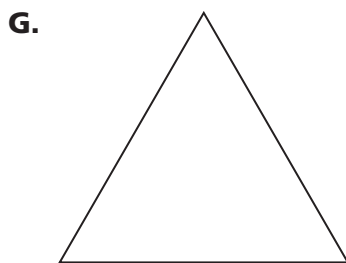
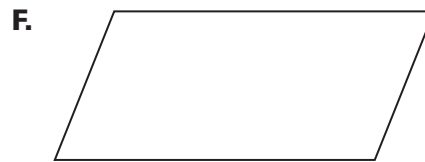
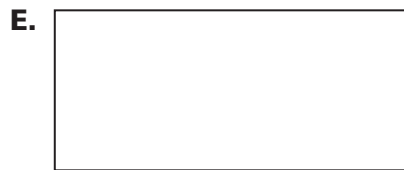
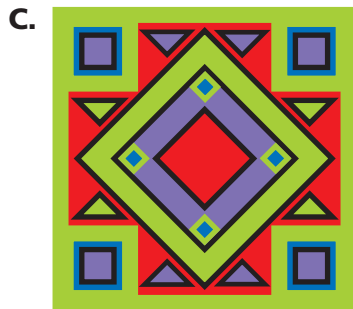
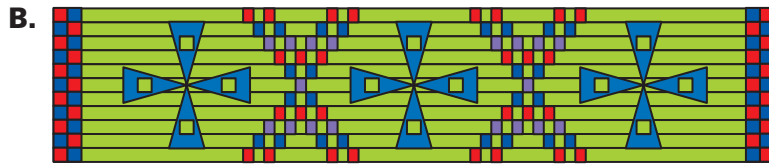
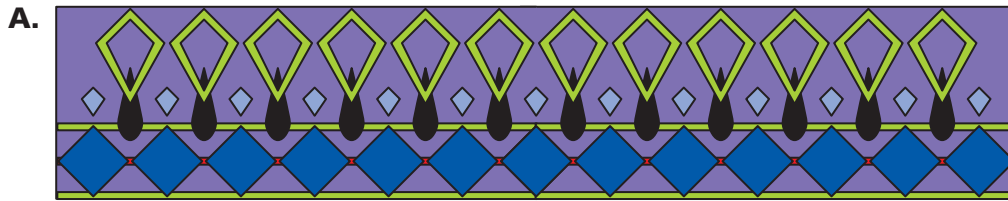


You can also use tracing paper to check for reflection symmetry. Trace the figure and the possible line of symmetry. Then reflect the tracing over the possible line of symmetry. If the reflected tracing fits exactly on the original figure, the figure has reflection symmetry.

*What happens to the line of symmetry when you reflect the tracing and match it with the original figure? Does its location change?*

## Problem 1.1 Reflection Symmetry

Use a mirror, tracing paper, or other tools to find all lines of symmetry in each design or figure.

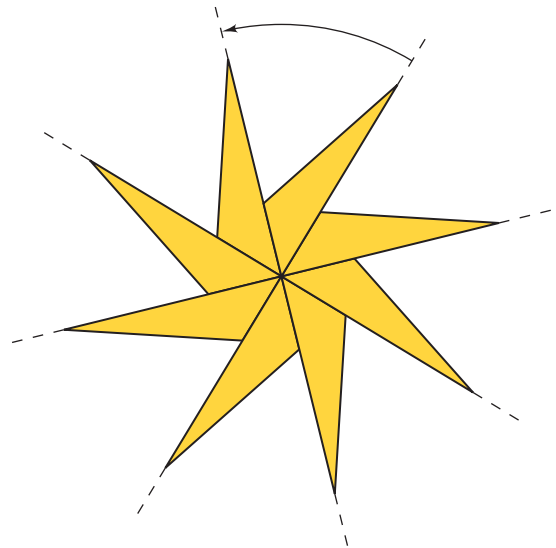


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## 1.2 Rotation Symmetry

The pinwheel design at the right does not have reflection symmetry. However, it can be turned less than a full turn around its center point in a counterclockwise direction to positions in which it looks the same as it does in its original position. Figures with this property are said to have **rotation symmetry**.

The windmill, snowflake, and wagon wheel pictured below also have rotation symmetry.



Which two of the three objects pictured above also have reflection symmetry?

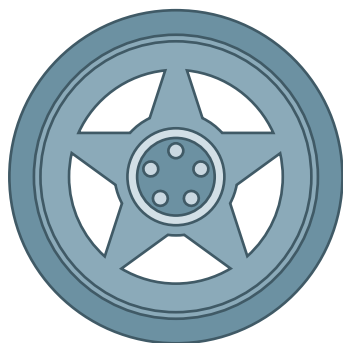
To describe the rotation symmetry in a figure, you need to specify two things:

- The *center of rotation*. This is the fixed point about which you rotate the figure.
- The *angle of rotation*. This is the *smallest* angle through which you can turn the figure in a counterclockwise direction so that it looks the same as it does in its original position.

There are several rotation angles that move the pinwheel design above to a position where it looks like the original. In this problem, you will consider how these angles are related to the angle of rotation.

## Problem 1.2 Rotation Symmetry

- A.** List all the turns of less than  $360^\circ$  that will rotate the pinwheel design to a position in which it looks the same as what is pictured. What is the angle of rotation for the pinwheel design?
- B.** In parts (1)–(3), list all the turns of less than  $360^\circ$  that will rotate the object to a position in which it looks the same as what is pictured. Then give the angle of rotation.
1. the windmill
  2. the snowflake
  3. the wagon wheel
- C.** Look at your answers for Questions A and B. For each object or figure, tell how the listed angles are related to the angle of rotation.
- D.** The hubcaps below have rotation symmetry. Complete parts (1) and (2) for each hubcap.



Hubcap 1



Hubcap 2

1. On a copy of the hubcap, mark the center of rotation. Then, find all the turns of less than  $360^\circ$  that will rotate the hubcap to a position in which it looks the same as what is pictured.
  2. Tell whether the hubcap has reflection symmetry. If it does, draw all the lines of symmetry.
- E.** Draw a hubcap design that has rotation symmetry with a  $90^\circ$  angle of rotation but no reflection symmetry.
- F.** Draw a hubcap design that has rotation symmetry with a  $60^\circ$  angle of rotation and at least one line of symmetry.
- G.** Investigate whether rectangles and parallelograms have rotation symmetry. Make sketches. For the shape(s) with rotation symmetry, give the center and angle of rotation.

**ACT** Homework starts on page 15.

**active math**  
**online**

For: Hubcap Maker  
Visit: PHSchool.com  
Web Code: apd-5102

## 1.3

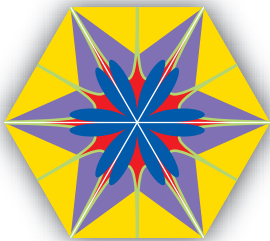
## Symmetry in Kaleidoscope Designs

A *kaleidoscope* (kuh ly duh skohp) is a tube containing colored beads or pieces of glass and carefully placed mirrors. When you hold a kaleidoscope up to your eye and turn the tube, you see colorful symmetric patterns.

The kaleidoscope was patented in 1817 by the Scottish scientist Sir David Brewster. Brewster was intrigued by the science of nature. He developed kaleidoscopes to simulate the designs he saw in the world around him.



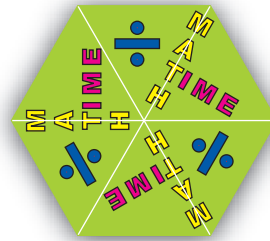
Five of the designs below are called *kaleidoscope designs* because they are similar to designs you would see if you looked through a kaleidoscope.



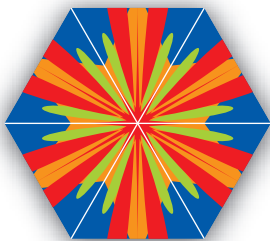
A



B



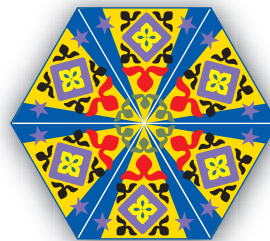
C



D



E

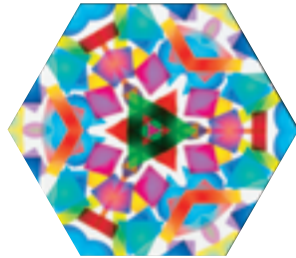


F

### Problem 1.3 Analyzing Symmetries

Use what you know about reflection and rotation symmetry to analyze the six designs.

- A. Locate all the lines of symmetry in the designs.
- B. Give the angles of rotation for the designs with rotation symmetry.
- C.
  1. Make a table showing the number of lines of symmetry and the angle of rotation for each design.
  2. What relationship, if any, do you see between the number of lines of symmetry and the angle of rotation?
  3. Analyze the kaleidoscope design below to see whether it confirms your relationship.

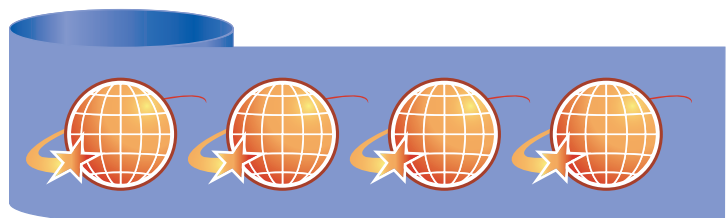


- D. Each of the designs can be made by repeating a small piece of the design. We call this piece the **basic design element**. For each design, sketch or outline the basic design element.
- E. One of the designs is *not* a kaleidoscope design. That is, it is not similar to a design you would see if you looked through a kaleidoscope. Which design do you think it is? Why?

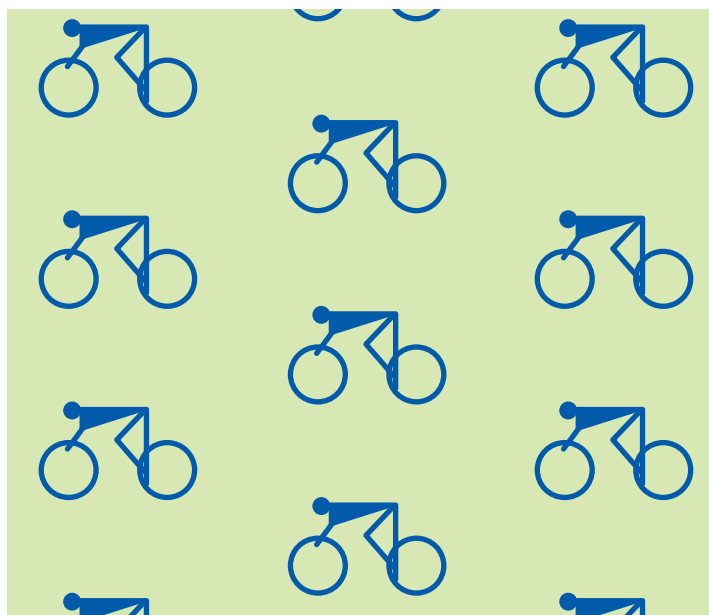
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## 1.4 Translation Symmetry

The next three designs are examples of “strip patterns.” You can draw a strip pattern by repeating a basic design element at regular intervals to the left and right of the original.



You can use a similar design strategy to make a “wallpaper pattern” like the one below.

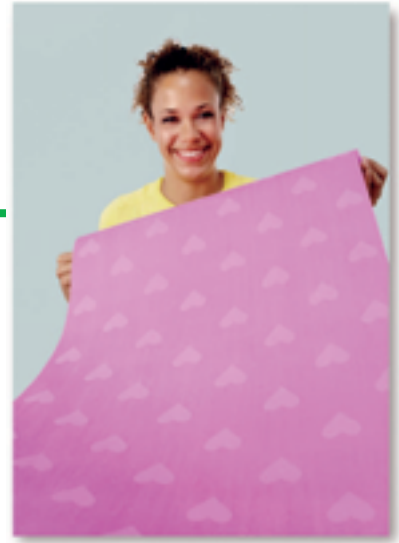




Making a strip pattern or a wallpaper design requires a series of “draw and move” steps. You draw a basic design element. Then, you slide your pencil to a new position and repeat the element. You slide in the same way to a new position and repeat the element again, and so on. The slide movements from one position to the next are called **translations**.

### Getting Ready for Problem 1.4

- Suppose the strip patterns on the previous page extend forever in both directions. Describe how you can move each infinite pattern so it looks exactly the same as it does in its original position.
- Suppose the wallpaper pattern on the previous page extends forever in all directions. Describe how you can move the infinite pattern so it looks exactly the same as it does in its original position.

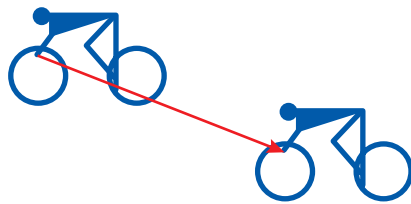


A design has **translation symmetry** if you can slide the whole design to a position in which it looks exactly the same as it did in its original position.

To describe translation symmetry, you need to specify the distance and direction of the translation. You can do this by drawing an arrow indicating the slide that would move the design “onto itself.”

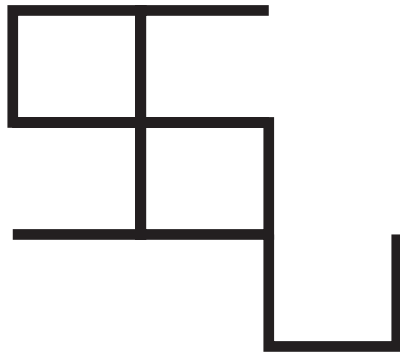
Questions about translation symmetry are of two kinds.

- Given a basic design element, how can you use draw-and-slide operations to produce a pattern with translation symmetry?
- How can you tell whether a given design has translation symmetry?

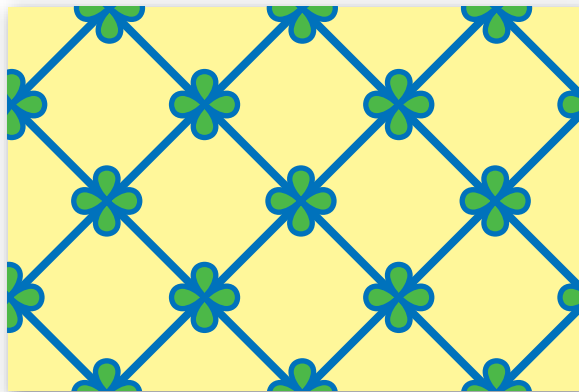


## Problem 1.4 Translation Symmetry

- A. Cut a long strip of paper about one inch wide. Use the basic design element below to draw a strip pattern on the paper. The resulting strip pattern can be found in fabrics made by the Mayan people who live in Central America.



- B. 1. Below is a part of a design that extends forever in all directions. Outline a basic design element that can be used to make the entire design using only translations.



2. Describe precisely how the basic design element can be copied and translated to produce the pattern. Your description should include diagrams with arrows and measures of distances.

**ACE** Homework starts on page 15.