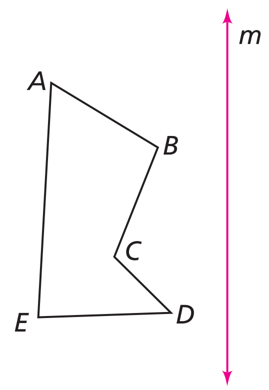
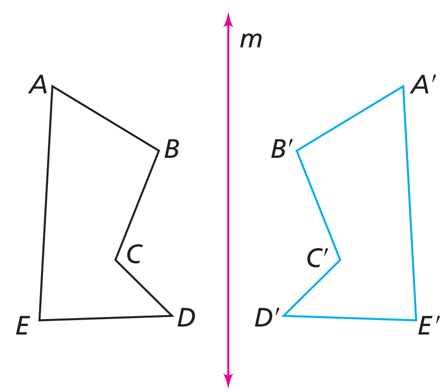
Investigation 2

Symmetry Transformations

You can make symmetric designs by copying a basic figure to produce a balanced pattern. For example, to construct a design with reflection symmetry, start with pentagon $ABCDE$ and line m below.



Reflect $ABCDE$ in line m to get pentagon $A'B'C'D'E'$.



Reflecting a figure in a line is an example of a geometric operation called a **transformation.** A transformation produces a copy, or *image,* of an original figure in a new position.

In this investigation, you will explore the transformations associated with reflection, rotation, and translation symmetry.

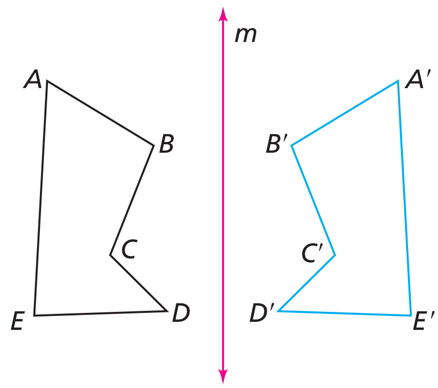
2.1 Describing Line Reflections

Transformations that produce patterns with reflection symmetry are called **line reflections.**

Suppose you start with pentagon $ABCDE$ and line m from the previous page.

How can you locate the reflection image $A'B'C'D'E'$ without folding, tracing, or using a mirror?

Look for a precise way to describe a line reflection as you work through this problem.

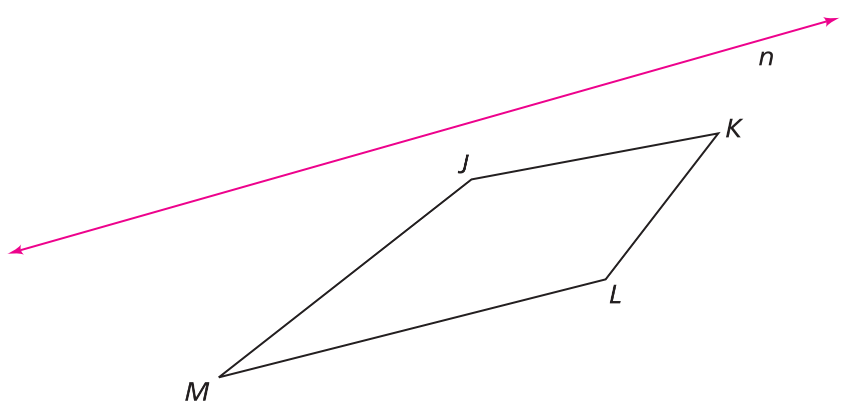


Problem 2.1 Describing Line Reflections

A. Copy pentagon $ABCDE$, its image, $A'B'C'D'E'$, and the line of reflection, m.

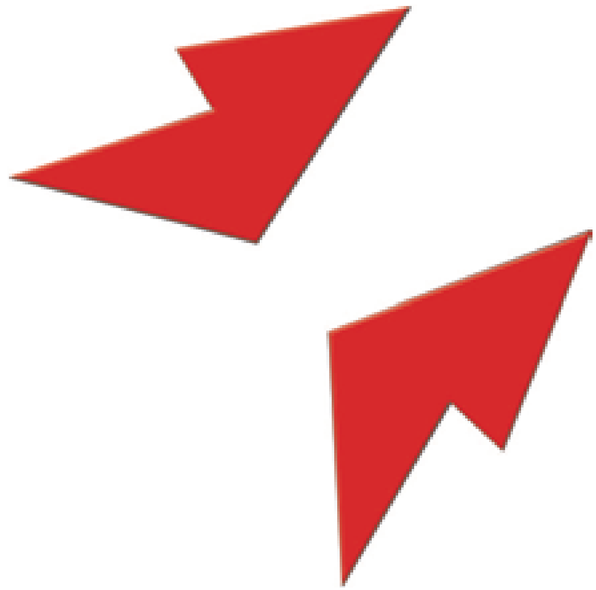
1. Draw segments connecting each vertex of $ABCDE$ to its image on $A'B'C'D'E'$. In other words, connect A to A', B to B', and so on.
2. Use tools for measuring angles and lengths to see how the line of reflection is related to each segment you drew in part (1).
3. Describe the patterns in your measurements from part (2).

B. 1. Copy quadrilateral $JKLM$ and line m below. Use what you discovered in Question A to draw $J'K'L'M'$, the image of $JKLM$ under a reflection in line n. Use only a pencil, a ruler, and an angle ruler or protractor. Explain how you located the image.



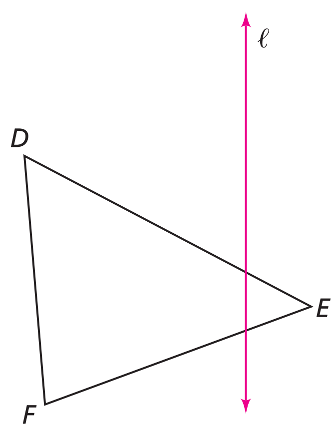
2. Does $JKLM$ have any symmetries? Explain.
3. Does the figure made up of both $JKLM$ and its reflection, $J'K'L'M'$, have any symmetries? Explain.

C. The design below has reflection symmetry. Copy the design. Use only a pencil, a ruler, and an angle ruler or protractor to locate the line of symmetry. Explain how you found the location of the line.



D. Complete this definition: A line reflection in a line m matches each point X on a figure to an image point X' so that . . .

E. Copy triangle DEF and line ℓ. Notice triangle DEF crosses the line.



1. Does triangle DEF have reflection symmetry?
2. Draw the image of triangle DEF under a reflection in line ℓ.
3. Does the final figure, made of triangle DEF and its image, have reflection symmetry? Explain.

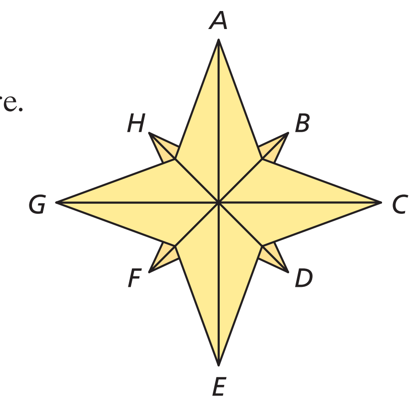
F. When you reflect a figure in a line, you can visualize reflecting the entire plane and taking the figure along for the ride. Are any points in the plane unmoved by a reflection? That is, are there any fixed points? Explain.

active math online
For: The Transformation Tool
Visit: PHSchool.com
Web Code: apd-5201

ACE **Homework starts on page 36.**

2.2 Describing Rotations

The compass star shown at the right has rotation symmetry. You can turn it around its center point to a position in which it looks identical to the original figure. Such a turn matches each point in the original to an image point on the original figure.



The transformation that turns a figure about a point, matching each point to an image point, is called a **rotation.**

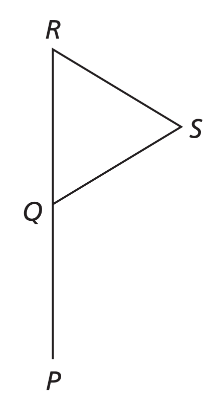
In this problem, look for a way to describe the relationship between any point X and its image point X' under a rotation.

Problem 2.2 Describing Rotations

A. Copy the compass star above.

1. What is the smallest counterclockwise turn (in degrees) that will rotate the star to a new position in which it looks identical to the original?
2. Because the original figure has rotation symmetry, the image of each point on the original star is also a point on the rotated star. List the pairs of points and their images, matched by the rotation in part (1).
3. Describe the paths the points of the original figure follow as they are "moved" to the positions of their images.
4. How would you describe the relationship among any point X, its image, and the center of the compass star?

B. Copy the "flag" at the right.



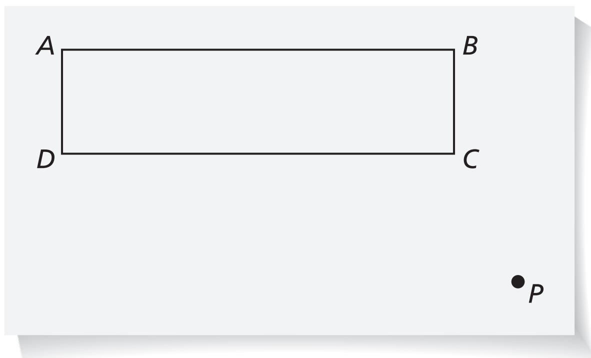
1. Does the flag have rotation symmetry? Explain.
2. Draw the flag's image, $PQ'R'S'$, after a 60° counterclockwise rotation about point P. Use only tools for drawing and measuring segments and angles. Explain how you located the image points.
3. Does the final figure, made up of the original flag and its image, have rotation symmetry?

4. For which of these rotations about point P will the original flag and its image form a design with rotation symmetry?

   12°   90°   40°   45°   180°

5. Can you make a design with rotation symmetry about P that consists of the original flag and more than one rotation image? If so, tell what rotations of the original are needed. If not, explain why.

C. **1.** Point P is outside of rectangle $ABCD$. Copy the rectangle and point P. Draw the image of $ABCD$ after a 90° counterclockwise rotation about point P. Use only tools for drawing and measuring segments and angles.



2. On your drawing in part (1), use a compass to draw a circle with center P and radius PB.

3. Describe the path the image of vertex B travels in a 90° rotation about point P. How is the movement of the image of vertex A similar? How is it different?

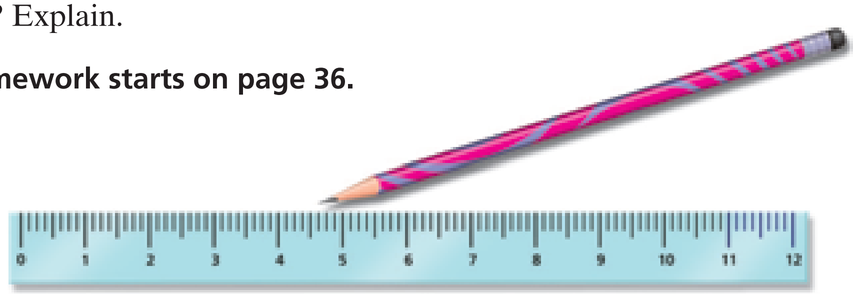
4. What can you say about segments PA and PA'? What can you say about segments PB and PB'?

5. Find the measures of angles APA', BPB', CPC', and DPD'. What can you conclude?

D. Complete this definition: A rotation of d degrees about a point P matches any point X with an image point X' so that . . .

E. When you rotate a figure about a point, you can visualize rotating the entire plane and taking the figure along for the ride. Are any points in the plane unmoved by a rotation? That is, are there any fixed points? Explain.

ACE **Homework starts on page 36.**



2.3 Describing Translations

Strip patterns and wallpaper designs have translation symmetry. You can slide the designs to new positions where the overall pattern appears unchanged. The transformation that slides a figure, matching each point to an image point, is called a **translation.**

In this problem, look for a way to describe the relationship between any point X and its image point X' after a translation.

Problem 2.3 Describing Translations

A. Copy Diagrams 1 and 2, which show polygon $GHJKLM$ and its images under two different translations. Do the following for each diagram:

- Label the vertices of the image G', H', J', K', L', and M' so that G' is the image of G, H' is the image of H, and so on.
- Draw line segments from each vertex of $GHJKLM$ to its image.
- Describe the pattern relating the segments GG', HH', etc.

Diagram 1

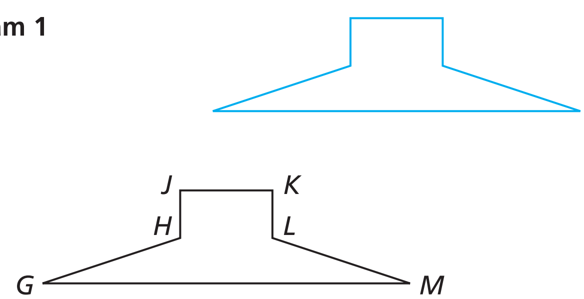
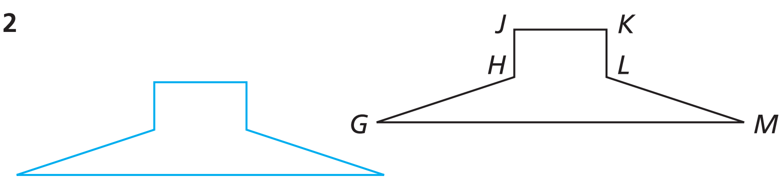
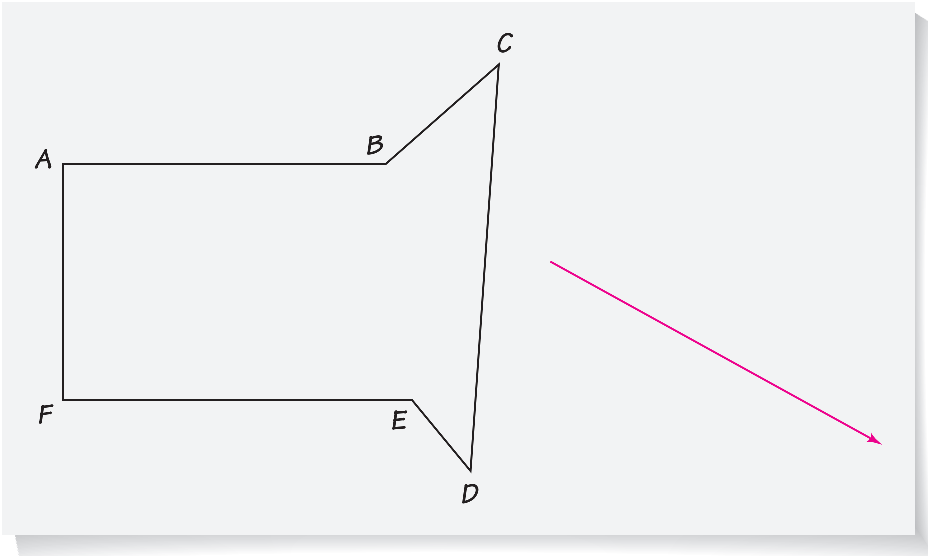


Diagram 2



B. Will drew a polygon and then drew an arrow to specify a translation. Copy the polygon and the arrow. Draw the image of the polygon under the translation.



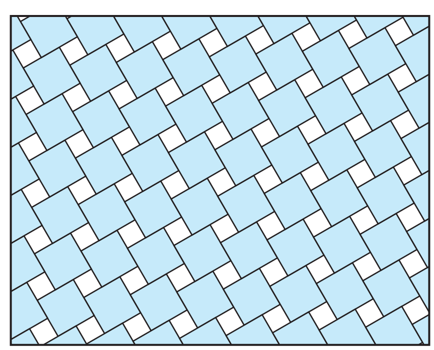
C. Complete this definition: A translation matches any two points X and Y with image points X' and Y' so that . . .

D. When you translate a figure, you can visualize translating the entire plane and taking the figure along for the ride. Are any points in the plane unmoved by a translation? That is, are there any fixed points? Explain.

ACE **Homework starts on page 36.**

2.4 Using Symmetry to Think About Tessellations

The design at the right is a **tessellation.** A tessellation is a design made from copies of a basic design element that cover a surface without gaps or overlaps.



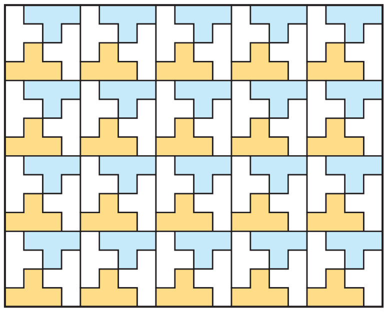
Can you spot the basic design element that repeats over and over?

What types of symmetry do you see in the design as a whole?

To decide if a basic element will tessellate, you need to investigate transformations of the basic design element to see if some combination of transformations will cover the entire surface.

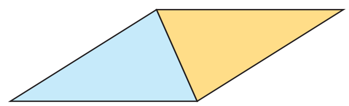
Problem 2.4 Using Symmetry to Think About Tessellations

A. The design below is a tessellation.

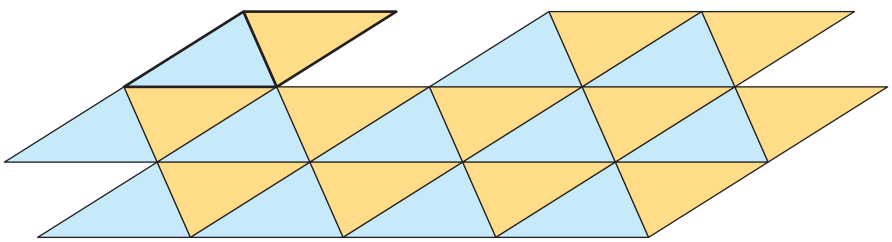


1. Sketch or outline the basic design element used to produce the tessellation. (**Hint:** Think about color as well as shape.)
2. Write directions or draw arrows to show how this basic element can be translated to produce other parts of the pattern.
3. Does the entire tessellation have reflection symmetry? Does it have rotation symmetry?

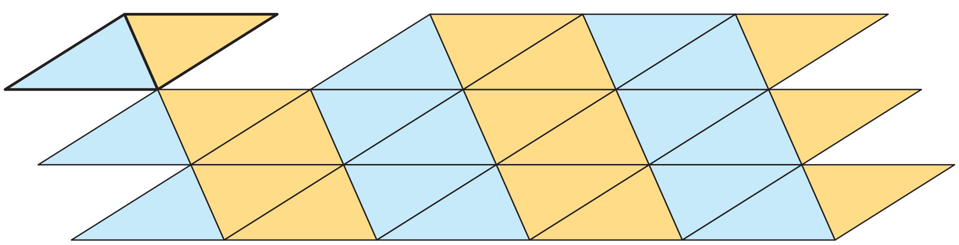
B. Rosslyn and Tevin both used the parallelogram at the right as a basic design element for a tessellation.



Rosslyn's Design



Tevin's Design



1. Does the basic design element have any symmetries? If so, describe them. If not, explain why.
2. Both Rosslyn and Tevin started their designs at the top left corner. Then, they made different moves to complete their tessellations. For each design, write detailed instructions explaining how to copy and move the highlighted element to fill the gap in the pattern. Is there more than one way to fill the gap?
3. Does either completed design have translation symmetry? Explain.
4. Does either completed design have reflection symmetry or rotation symmetry? Explain.

ACE **Homework starts on page 36.**

active math online
For: The interactive Escher-Like Maker
Web Code: apd-5204