Investigation 3

Exploring Congruence

In *Stretching and Shrinking,* you learned that figures with the same shape are similar. Figures that have the same size, as well as the same shape, are said to be **congruent.**

In this investigation, you will learn some ways to compare the size and shape of figures. You will also use your knowledge about congruence to solve geometry problems.

3.1 Relating Symmetry and Congruence

You have learned about three kinds of transformations that relate a figure to an image that is the same size and shape. You can use this idea to think about congruence. Two figures are congruent if you can reflect, rotate, or translate one figure so the final image fits exactly on top of the other figure.

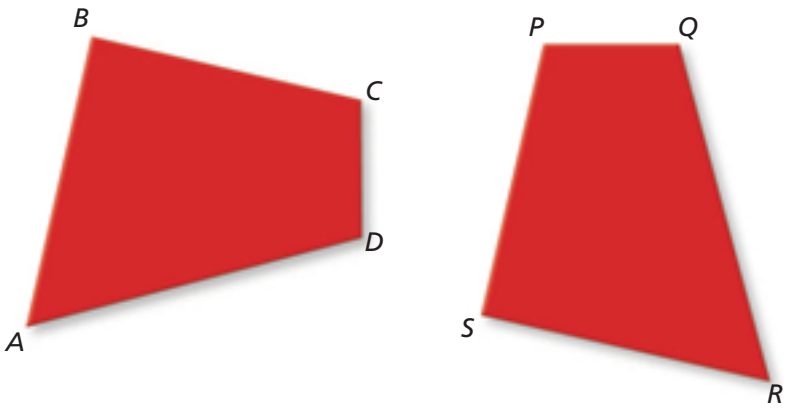
Getting Ready for Problem 3.1

All the turtles below are similar. The four small turtles are also congruent.

- What reflections, rotations, translations, and combinations of these transformations will relate each small turtle to the other congruent turtles?

Problem 3.1 Relating Symmetry and Congruence

The two quadrilaterals below are congruent.



A. Suppose you copied $ABCD$ and moved the copy so that it fit exactly on $PQRS$. Copy and complete these statements to show which vertices correspond. The arrow means "corresponds to."

$A \rightarrow$ ■ $B \rightarrow$ ■ $C \rightarrow$ ■ $D \rightarrow$ ■

B. The notation $\overline{AB}$ means "line segment AB." The symbol $\cong$ means "is congruent to." Copy and complete these statements to show which pairs of sides in the two quadrilaterals are congruent.

$\overline{AB} \cong$ ■ $\overline{BC} \cong$ ■ $\overline{CD} \cong$ ■ $\overline{DA} \cong$ ■

C. The notation $\angle A$ means "angle A." Copy and complete these statements to show which angles are congruent.

$\angle A \cong$ ■ $\angle B \cong$ ■ $\angle C \cong$ ■ $\angle D \cong$ ■

D. Make a copy of $ABCD$ on tracing paper. Investigate combinations of reflections, rotations, and translations that will move the copy of $ABCD$ exactly on top of $PQRS$.

1. Is there a single reflection, rotation, or translation that matches each point of $ABCD$ onto its corresponding point on $PQRS$?
2. What combination of reflections, rotations, and translations matches each point of $ABCD$ onto its corresponding point on $PQRS$?

E. How could you rename $PQRS$ so that the name shows how the vertices of $PQRS$ correspond to those in $ABCD$?

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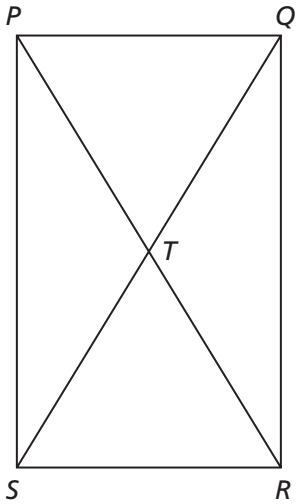
3.2 Congruent Triangles

Congruent polygons are often used to enhance the beauty and strength of buildings and other structures. This photograph of the George Washington Bridge in New York City shows congruent triangles in the bridge's towers.

If you look closely at the photograph, you can see that each tower is made up of rectangles. The diagonals of these rectangles form triangles.

Problem 3.2 Congruent Triangles

The figure at the right shows how the pieces of the bridge tower fit together.



A. How many triangles can you find in the figure? Make a list using names such as $\triangle PTQ$, $\triangle PTS$, and so on. (The notation $\triangle PTQ$ means "triangle PTQ.")

B. Describe the symmetries of rectangle $PQRS$.

C. Make a list of the pairs of triangles that appear to be congruent.

D. For each pair you listed in Question C, describe a reflection, rotation, or translation that would match one triangle to its congruent partner.

E. Zoe writes $\triangle PTQ \cong \triangle RTS$. Is she correct? What information do you need to check her claim?

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3.3 The Matching Game

The Matching Game will help you learn about properties of congruent polygons. The game is played by two teams of two players. The players can use only tools for drawing and measuring segments and angles.

- Each team has 15 points to "spend."
- One player on the first team is given a polygon with side and angle measures shown.
- The player gives his or her partner directions for drawing a congruent copy of the polygon, but does not show the polygon to his or her partner. For each side length or angle measure mentioned in the directions, the team must spend 1 point.
- The directions may include the words "triangle" or "quadrilateral" or "pentagon," but not any other clues such as "regular" or "right" or "square."
- When the drawer believes he or she has made a congruent copy, the players say "Done." The other team checks to see if the drawing is the same size and shape as the original.
- The teams take turns. The first team to spend all 15 of its points loses.

Getting Ready for Problem 3.3

How would you give directions to a partner for drawing a congruent copy of Figure 1?

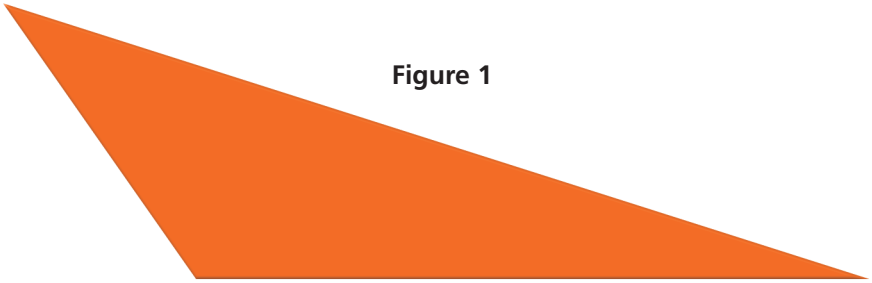


Figure 1

Use a set of *Matching Game* figures to play the game. Keep an eye out for winning strategies.

To play *The Matching Game* well, you need to think about this question:

How much information about two polygons will guarantee that they are congruent?

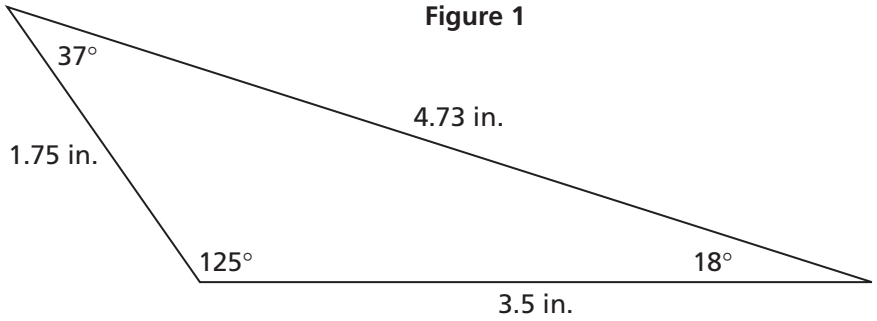
Problem 3.3 Congruent Triangles

A. 1. Suppose you are playing *The Matching Game*. Add steps to complete these drawing directions to make a figure congruent to Figure 1.

Step 1 Draw $\angle X$ with a measure of 125°.

Step 2 On one side of $\angle X$, mark a point Y so that $\overline{XY}$ is 3.5 inches long.

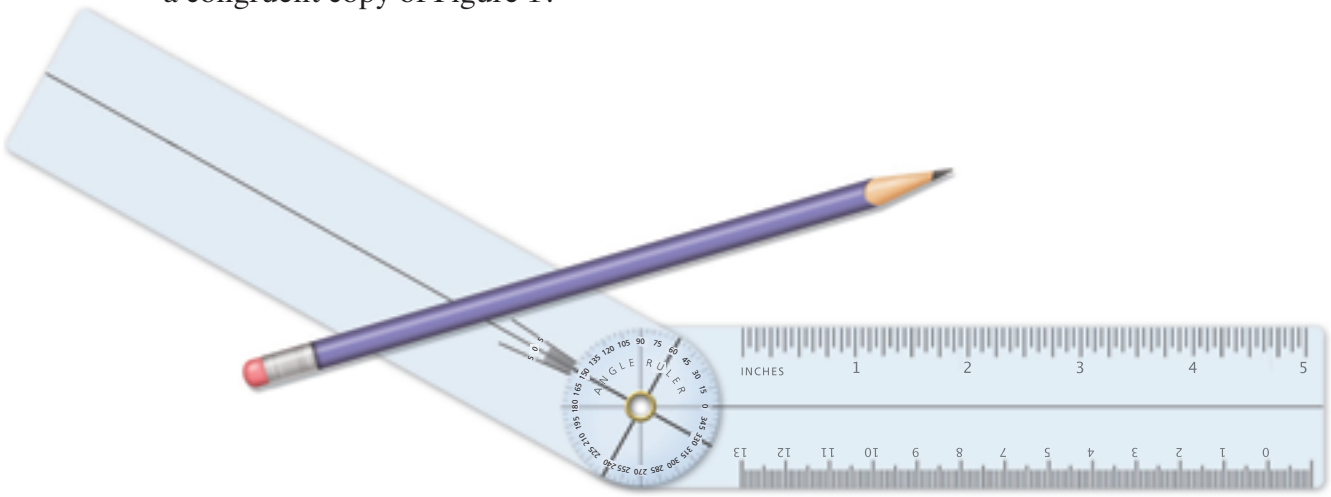
Figure 1



2. How many points would your team spend for all of your steps?

3. Find some other lists of directions that would work. Give the number of points for each possibility.

4. What are the fewest points your team would need to spend to make a congruent copy of Figure 1?



B. Tell whether you would be certain to draw a congruent copy of Figure 2 if you followed each set of directions.

Figure 2

Direction Set 1
Step 1 The figure is a triangle.
Step 2 Draw $\angle A$ with a measure of 35°.
Step 3 Draw $\angle B$ with a measure of 45°.
Step 4 Draw $\angle C$ with a measure of 100°.

Direction Set 2
Step 1 The figure is a triangle.
Step 2 Draw $\angle B$ with a measure of 45°.
Step 3 Draw $\overline{AB}$ with a length of 5 inches.
Step 4 Draw $\angle C$ with a measure of 100°.

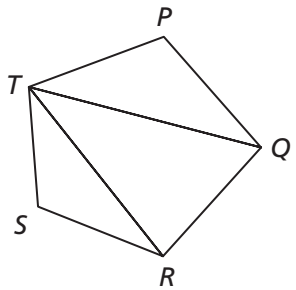
Direction Set 3
Step 1 The figure is a triangle.
Step 2 Draw $\angle B$ with a measure of 45°.
Step 3 Draw $\overline{AB}$ with a length of 5 inches.
Step 4 Draw $\overline{AC}$ with a length of 3.6 inches.

Direction Set 4
Step 1 The figure is a triangle.
Step 2 Draw $\overline{AB}$ with a length of 5 inches.
Step 3 Draw $\overline{BC}$ with a length of 2.9 inches.
Step 4 Draw $\overline{CA}$ with a length of 3.6 inches.

C. Compare your results for Questions A and B with those of other students. Look for patterns that will help you answer this question:

What is the minimum information you need about the sides and angles of two triangles to be sure those triangles are congruent?

D. Identify a minimum set of angles and segments you could measure to give your partner enough information to draw a congruent copy of the pentagon. (You do not have to find the measures.)



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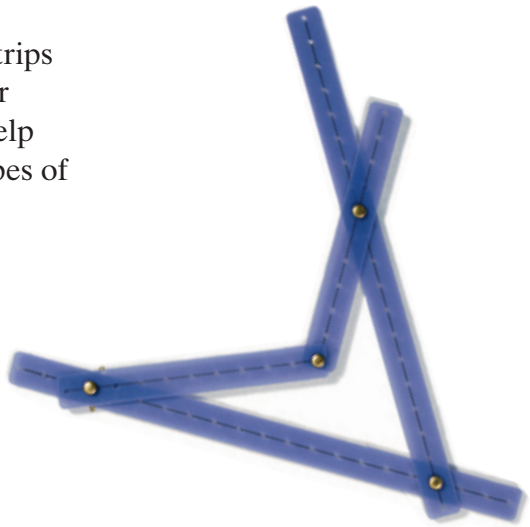
3.4 Polystrip Triangles and Quadrilaterals

In earlier geometry units, you used polystrips to build triangles, quadrilaterals, and other polygons. Building polystrip figures can help you study how side lengths affect the shapes of polygons.



Problem 3.4 Side Lengths and Congruence

Use polystrips to explore the questions below. Make sketches to support your answers.

A. Tell how many different (non-congruent) triangles you can construct with each set of side lengths.

1. 20 cm, 20 cm, and 10 cm

2. 10 cm, 15 cm, and 20 cm

3. 20 cm, 5 cm, and 10 cm

4. 10 cm, 10 cm, and 10 cm

B. Tell how many different quadrilaterals you can construct with each set of side lengths in the given order.

1. 10 cm, 20 cm, 10 cm, 20 cm

2. 20 cm, 5 cm, 5 cm, and 5 cm

3. 10 cm, 10 cm, 20 cm, 20 cm

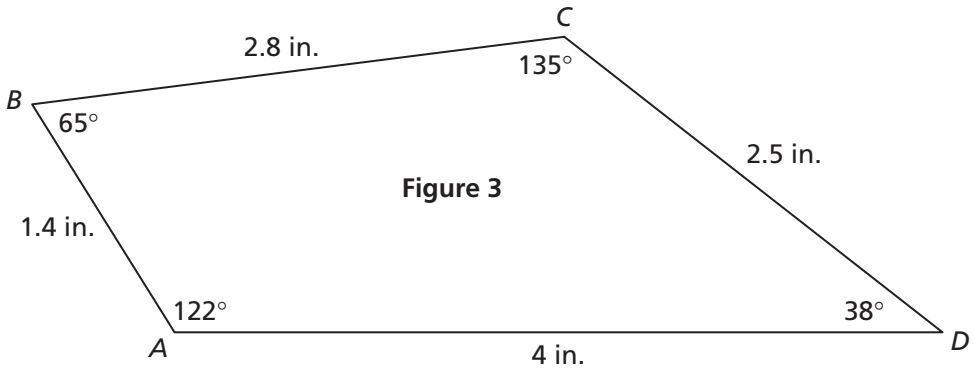
C. Compare your results for Questions A and B with those of other students. Look for patterns that will help you answer these questions.

1. What does knowing only the side lengths tell you about the size and shape of a triangle or a quadrilateral?

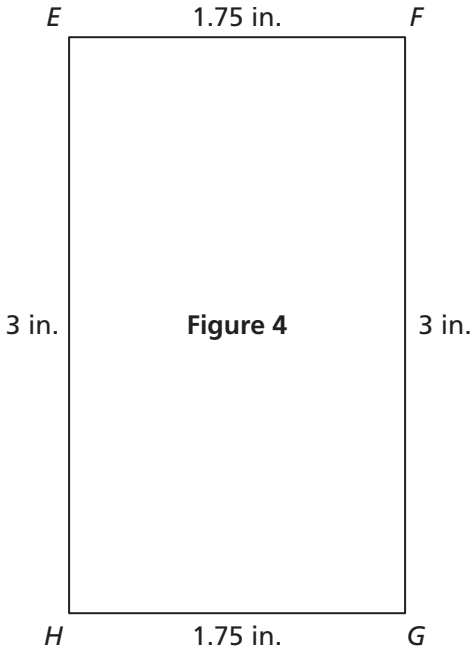
2. Suppose the corresponding sides of two triangles are the same length. Can you conclude that the triangles are congruent?

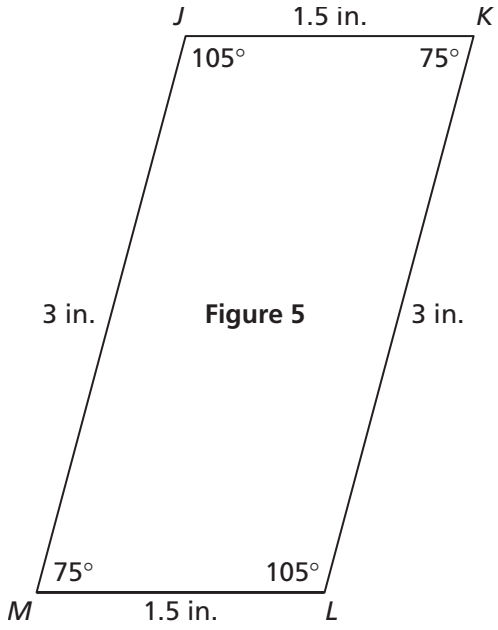
3. Suppose the corresponding sides of two quadrilaterals are the same length. Can you conclude that the quadrilaterals are congruent?

D. Figure 3 from *The Matching Game* is shown below. Write a set of drawing directions for the figure. How many points would you spend to give your directions?



E. Figures 4 and 5 from *The Matching Game* are shown below. For each quadrilateral shape, write a set of drawing directions that would cost your team the minimum number of points. Do not use the name of the figure (rectangle or parallelogram) in your directions.





F. Compare your results for Questions D and E with those of other students. Look for patterns that will help you answer this question:

What is the minimum information you need about the sides and angles of two quadrilaterals to be sure those quadrilaterals are congruent?

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