Investigation 4

Applying Congruence and Symmetry

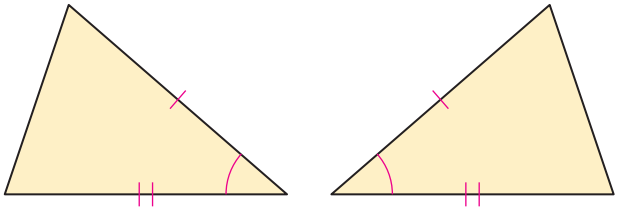
In Investigation 3, you explored two questions:

What does it mean to say that two geometric figures are congruent?

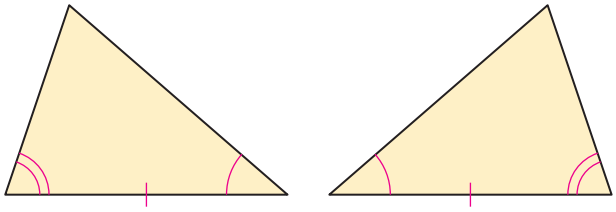
What information do you need to decide whether two triangles or two quadrilaterals are congruent?

You discovered that two triangles are congruent if any of the following are true:

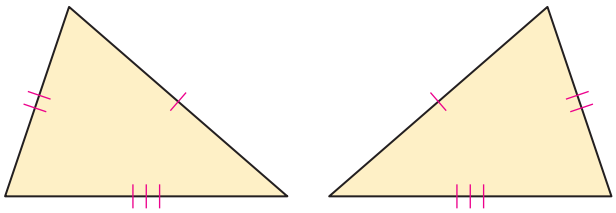
- Two sides and the included angle of one triangle are congruent to two sides and the included angle of another.



- Two angles and the common side in one triangle are congruent to two angles and the common side in another.



- Three sides of one triangle are congruent to three sides of another.

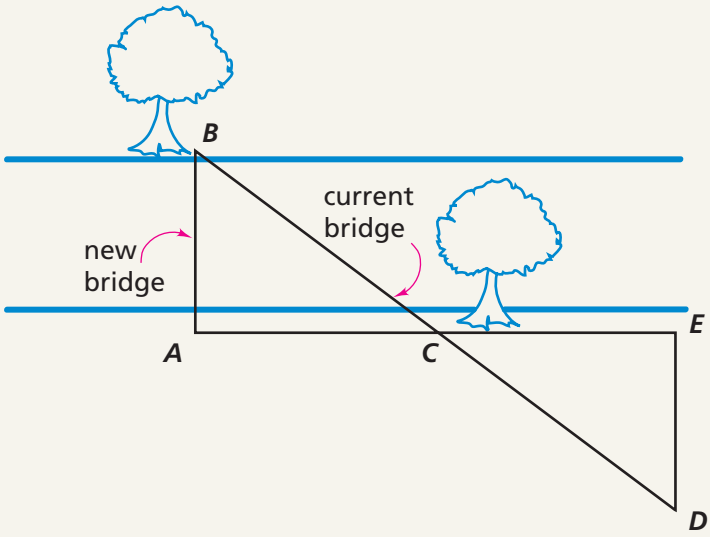


In this investigation, you will apply what you have learned about congruent polygons, symmetry, and symmetry transformations to solve problems.

4.1 Finding Distances Without Measuring

An engineer is planning a footbridge across a river. Currently, there is an old cable bridge strung between two trees on opposite sides of the river (points B and C). The engineer wants the new bridge to start at one of the trees (point B) and span the shortest distance across the river (to point A). She needs to figure out how long the new bridge must be.

Her notebook includes the following sketch and instructions:



Take the old cable bridge down. Lay the old bridge on the ground from point C to point D so that points B, C, and D lie in a straight line.

Place a stake at point A directly across the river from point B.

Place another stake at point E so that points A, C, and E lie in a straight line.

Problem 4.1 Finding Distances Without Measuring

Use the engineer's sketch and notes to answer these questions.

A. In the two triangles, identify the sides and angles you know are congruent.

B. Do the engineer's notes provide enough information to conclude that the two triangles are congruent? If not, what additional information does she need?

C. Assume that the additional information you described in Question B is true. How could the engineer use the congruence of the triangles to find the distance across the river?

D. Which transformation or combination of transformations would match one of the triangles to the other?

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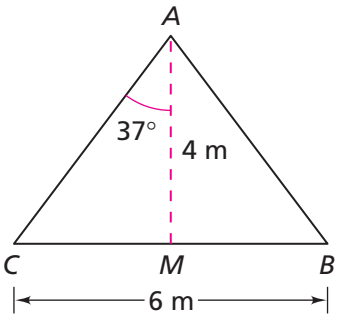
4.2 Using Symmetry to Find Properties of Shapes

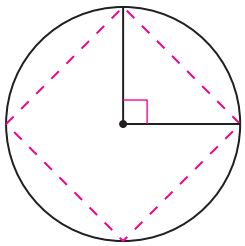
You can sometimes use the symmetry in geometric figures to find information about sides and angles of these figures.

Problem 4.2 Using Symmetry to Find Properties of Shapes

A. $\overline{AM}$ is on a line of symmetry for $\triangle ABC$. Some lengths and angle measures are given. Find the other lengths and angle measures.

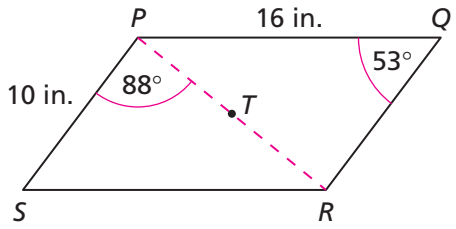


B. **1.** In the circle below, two radii form the legs of a right triangle. The hypotenuse of the triangle is a dashed segment. What transformations of the hypotenuse would generate the quadrilateral formed by the dashed segments? Give more than one possibility if you can.



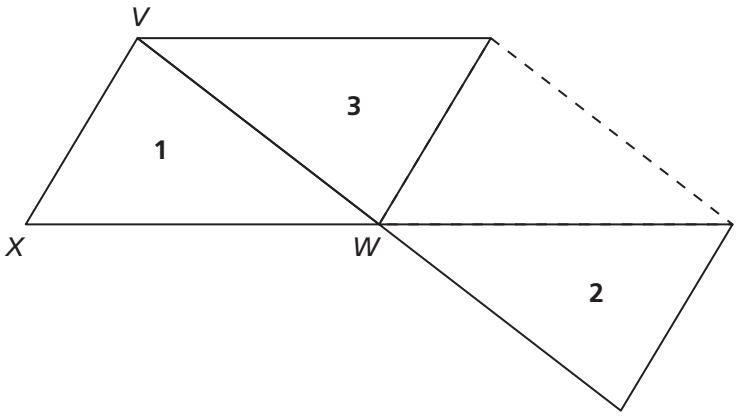
2. What shape is formed if the transformations you found in part (1) are applied to the full triangle? How do you know? Give reasons to support your answer.

C. 1. Quadrilateral $PQRS$ has rotation symmetry about point T, the midpoint of $\overline{PR}$, with a 180° angle of rotation. Find the missing lengths and angle measures.



2. Use the given information and the information you found in part (1) to write a convincing argument that quadrilateral $PQRS$ is a parallelogram.

D. Nathalie is trying to use transformations to show that the sum of the angle measures of a triangle is 180°. She starts with $\triangle VWX$ below (Triangle 1) and applies some transformations to make Triangle 2, then Triangle 3. Here's what she has done so far.



1. Which transformations might Nathalie have performed so far?

2. Will a copy of $\triangle VWX$ exactly fit the gap with the dashed line? How do you know?

3. How does the finished figure show that the sum of the angle measures of $\triangle VWX$ is 180°?

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