Investigation 5

Transforming Coordinates

The drawing window in many computer geometry programs is a coordinate grid. You make designs by specifying the endpoints of line segments. When you transform a design, the coordinates of its points change according to specific rules.

In this investigation, you will explore transformations of figures on coordinate grids. You will write rules for transforming a point (x, y) to its image under translations, rotations, and reflections. You will also look at the results of combining transformations. What you see and do visually is tracked and checked as a change in algebraic symbols.

5.1 Coordinate Rules for Reflections

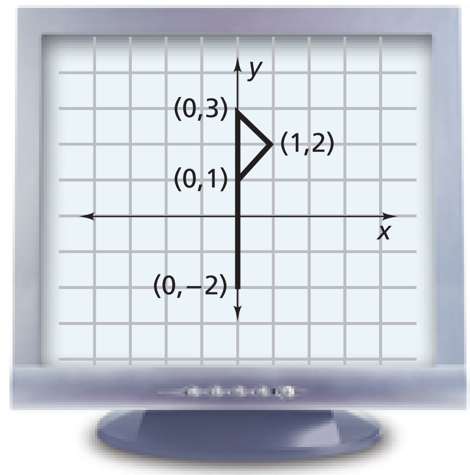
The flag shape below consists of three segments. It was produced in a computer program, using the commands shown. The commands tell the computer to draw segments between the specified endpoints.

Draw:

Line [(0, −2), (0, 3)]

Line [(0, 3), (1, 2)]

Line [(1, 2), (0, 1)]



Getting Ready for Problem 5.1

- Is there a different set of commands that will produce the same flag as the one above?
- What commands will produce a square centered at the origin?
- What commands will produce a non-square rectangle?

Most geometry software allows you to reflect, rotate, and translate figures. These transformations change the coordinates of the figure.

In this problem, you will explore reflections of figures on coordinate grids. By looking for patterns in your results, you will be able to write algebraic rules for reflecting any point (x, y).

Problem 5.1 Coordinate Rules for Reflections

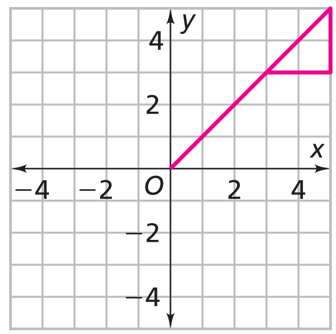
A. **1.** Copy and complete these commands for drawing the flag.

Draw:

Line [(■, ■), (■, ■)]

Line [(■, ■), (■, ■)]

Line [(■, ■), (■, ■)]

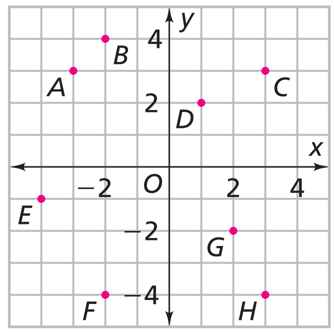


2. Write commands that will draw the image of the original flag under a reflection in the y-axis. Describe the pattern that relates each point (x, y) to its image. Which points remain unchanged?

3. Write commands that will draw the image of the original flag under a reflection in the x-axis. Describe the pattern that relates each point (x, y) to its image. Which points remain unchanged?

4. Write commands that will draw the image of the original flag under a reflection in the line $y = x$. Describe the pattern relating points (x, y) to their image. Which points remain unchanged?

Make copies of the diagram below as needed for Questions B–E.



B. List the coordinates of points A–H.

C. 1. List the coordinates of the images of points A–H under a reflection in the y-axis. Label the image points A'–H'.

2. Compare the coordinates of each original point with the coordinates of its image. Use the patterns you see to complete this rule for finding the image of any point (x, y) under a reflection in the y-axis:

$(x, y) \rightarrow (\blacksquare, \blacksquare)$

D. 1. List the coordinates of the images of points A–H under a reflection in the x-axis. Label the image points A''–H''.

2. Compare the coordinates of each original point with the coordinates of its image. Complete this rule for finding the image of any point (x, y) under a reflection in the x-axis:

$(x, y) \rightarrow (\blacksquare, \blacksquare)$

E. 1. List the coordinates of the images of points A–H under a reflection in the line $y = x$. Label the image points A'''–H'''.

2. Compare the coordinates of each original point with the coordinates of its image. Complete this rule for finding the image of any point (x, y) under a reflection in the line $y = x$:

$(x, y) \rightarrow (\blacksquare, \blacksquare)$

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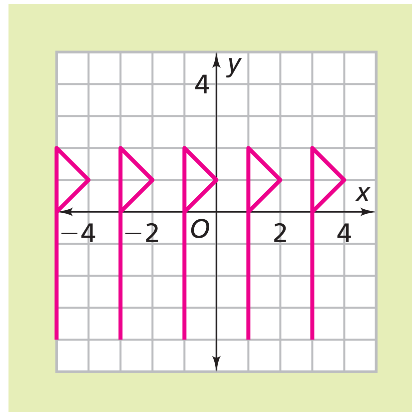
5.2 Coordinate Rules for Translations

In this problem, you will explore how translating a figure affects its coordinates.

Problem 5.2 Coordinate Rules for Translations

A. In the design at the right, the left-most flag was made with the commands shown.



Draw:

Line [(−5, −4), (−5, 2)]

Line [(−5, 2), (−4, 1)]

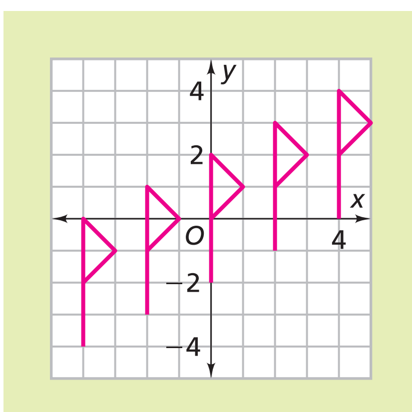
Line [(−4, 1), (−5, 0)]

1. Write commands for drawing the other four flags. Each set of commands should draw the segments in the same order as the commands for the original design.

2. Compare the commands for the five flags. Describe a pattern that relates the coordinates of each flag to the coordinates of the flag to its right.

3. Describe a pattern that relates the coordinates of each flag to the coordinates of the flag to its left.

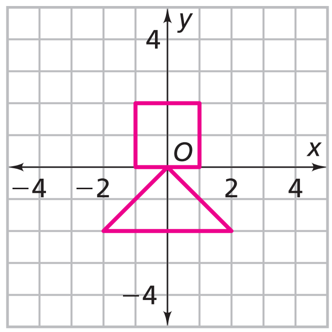
B. 1. Write a set of commands for drawing the left-most flag in the design below. Then write commands for drawing the other four flags.



2. Compare the commands for the five flags. Describe a pattern that relates the coordinates of each flag to the coordinates of the flag to its right.

3. Describe a pattern that relates the coordinates of each flag to the coordinates of the flag to its left.

C. 1. Copy the figure below. Experiment with translations of the figure. Try translations of different distances in directions parallel to the x-axis, y-axis, and the line $y = x$. In each case, tell what happens to the coordinates of a point on the original figure under the translation.



2. Complete each rule for finding the image of any point (x, y) under the given translation.

   a. Horizontal translation by b units: $(x, y) \rightarrow (■, ■)$

   b. Vertical translation by b units: $(x, y) \rightarrow (■, ■)$

   c. Translation by b units in the direction of the line $y = x$: $(x, y) \rightarrow (■, ■)$

D. In parts (1)–(5), tell whether the rule describes a translation of any original figure.

1. $(x, y) \rightarrow (x + 2, y - 3)$
2. $(x, y) \rightarrow (2x, y)$
3. $(x, y) \rightarrow (x + 1, 3y)$
4. $(x, y) \rightarrow (x - 2, y + 1)$
5. For each rule that describes a translation, describe in words the image made by applying the translation to all points on the original figure.

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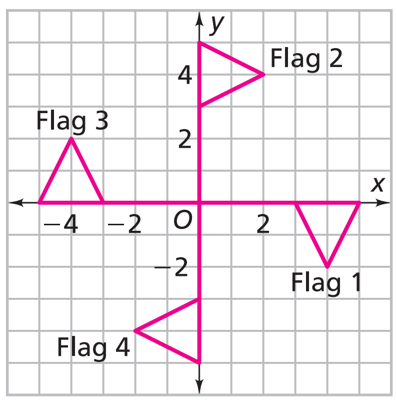
5.3 Coordinate Rules for Rotations

You have explored rules for reflecting and translating a point (x, y). Writing rules for rotations is more difficult. In this problem, you will explore a few simple cases.

Getting Ready for Problem 5.3

Think of rotating the four flags in this diagram 90° counterclockwise about the origin (0, 0).



- How do the coordinates of a point on Flag 1 compare with their image points under the 90° rotation? Try some points to see. Record your results.
- How do the points on Flags 2, 3, and 4 compare with their image points under the same rotation? Test some points and record your results.
- Do you see a pattern that you could use to write a coordinate rule for a 90° rotation about the origin?

Problem 5.3

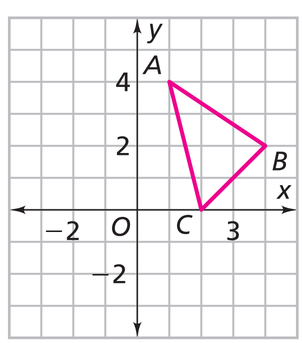
A. 1. Copy and complete these drawing commands for $\triangle ABC$.

Draw:

Line [(■, ■), (■, ■)]

Line [(■, ■), (■, ■)]

Line [(■, ■), (■, ■)]



2. Write a set of commands that will draw the image of $\triangle ABC$ under each rotation about the origin.
   a. 90° counterclockwise rotation
   b. 180° rotation
   c. 270° counterclockwise rotation
   d. 360° rotation

B. 1. Organize your results from Question A as shown below.

Starting Point	90° Rotation	180° Rotation	270° Rotation	360° Rotation
$A(1, 4)$	(■, ■)	(■, ■)	(■, ■)	(■, ■)
$B(4, 2)$	(■, ■)	(■, ■)	(■, ■)	(■, ■)
$C(2, 0)$	(■, ■)	(■, ■)	(■, ■)	(■, ■)

2. Describe how the vertices of $\triangle ABC$ relate to the vertices of the image triangle.
3. Complete each rule for finding the image of any point (x, y) under the given rotation.
   a. 90° rotation about the origin: $(x, y) \rightarrow$ (■, ■)
   b. 180° rotation about the origin: $(x, y) \rightarrow$ (■, ■)
   c. 270° rotation about the origin: $(x, y) \rightarrow$ (■, ■)
   d. 360° rotation about the origin: $(x, y) \rightarrow$ (■, ■)

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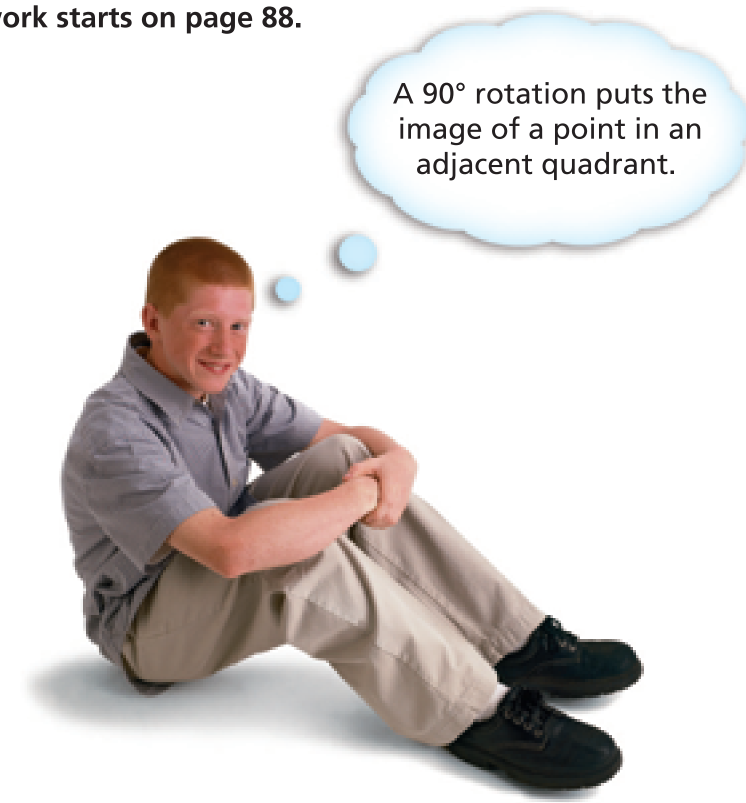


5.4 Coordinate Rules for Transformation Combinations

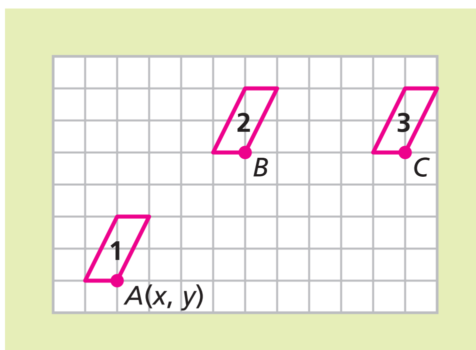
In some applications, like tessellations, an original shape may be transformed under a combination of transformations. In this problem you will explore how coordinate rules can track and check combinations of transformations.

Problem 5.4 Coordinate Rules for Transformation Combinations

A. 1. In the figure below, the distance between grid lines is 1 unit. Describe in words a transformation or a combination of transformations that will make images of the parallelogram as indicated.

a. Parallelogram 1 $\rightarrow$ Parallelogram 2

b. Parallelogram 1 $\rightarrow$ Parallelogram 3



2. In parts (a)–(b), refer to the rules for translations you wrote in Problem 5.2.

a. Suppose point A is translated in the direction of the line $y = x$ to point B. Then, this image point is translated horizontally to point C. Complete this rule to show these translations algebraically:

$$A \rightarrow B \rightarrow C$$

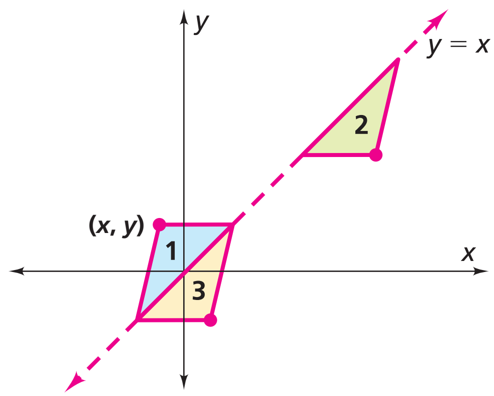
$$(x, y) \rightarrow (\blacksquare, \blacksquare) \rightarrow (\blacksquare, \blacksquare)$$

b. Suppose the translations in part (a) are reversed. In other words, the horizontal translation is applied to A and then the diagonal translation is applied to the image point. Complete this rule to show these translations algebraically:

$$(x, y) \rightarrow (\blacksquare, \blacksquare) \rightarrow (\blacksquare, \blacksquare)$$

c. In parts (a) and (b), you did the same translations, but in different orders. Was the final image the same in both cases? Do you think reversing the order of any two translations will give the same results?

B. 1. You can probably think of more than one way to transform triangle 1 below to make images at positions 2 and 3.



a. What coordinate rule rotates the point (x, y) by 180° about the origin?

b. What combination of coordinate rules first rotates the point (x, y) by 180° about the origin, and then translates the image diagonally in the direction of the line $y = x$ to position 2?

c. Suppose you reverse the order of the transformations in part (b). In other words, you first apply the translation and then apply the rotation. Is the image the same? Use coordinate rules to justify your answer.

2. Think about the two different orders of transformations you just applied to the triangle, a rotation then a translation or vice versa. Can you explain why the results are or are not the same by referring to the picture?

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