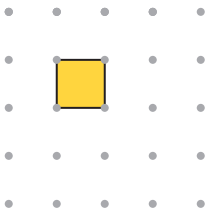
Investigation 2

Squaring Off

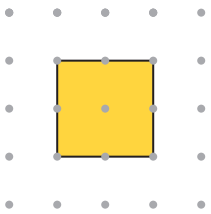
In this investigation, you will explore the relationship between the side lengths and areas of squares and use that relationship to find the lengths of segments on dot grids.

2.1 Looking for Squares

You can draw squares with different areas by connecting the points on a 5 dot-by-5 dot grid. Two simple examples follow.



Area = 1 square unit



Area = 4 square units

In this problem, you will explore other possible areas.

Problem 2.1 Looking for Squares

A. On 5 dot-by-5 dot grids, draw squares of various sizes by connecting dots. Draw squares with as many different areas as possible Label each square with its area. Include at least one square whose sides are not horizontal and vertical.

B. Analyze your set of squares and describe the side lengths you found.

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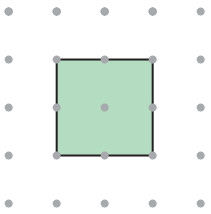
2.2 Square Roots

The area of a square is the length of a side multiplied by itself. This can be expressed by the formula $A = s \cdot s$, or $A = s^2$.

If you know the area of a square, you can work backward to find the length of a side. For example, suppose a square has an area of 4 square units. To find the length of a side, you need to figure out what positive number multiplied by itself equals 4. Because $2 \cdot 2 = 4$, the side length is 2 units. We call 2 a **square root** of 4.



This square has an area of 4 square units. The length of each side is the square root of 4, or 2, units.

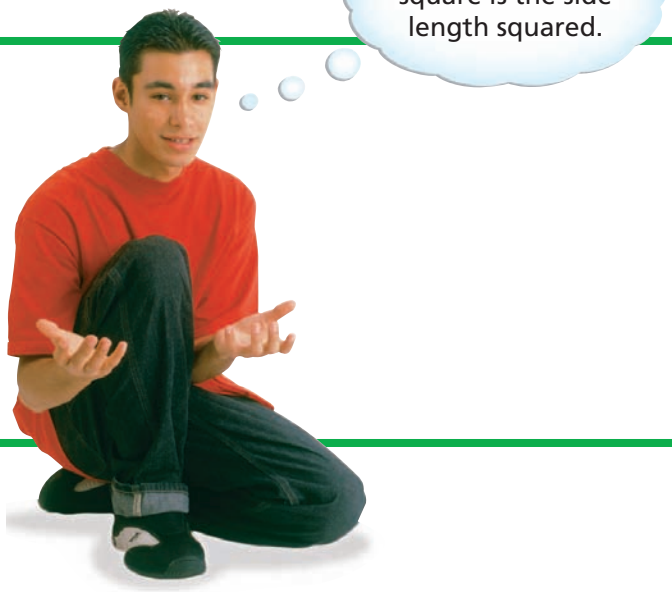
In general, if $A = s^2$, then s is a square root of A. Because $2 \cdot 2 = 4$ and $-2 \cdot -2 = 4$, 2 and -2 are both square roots of 4. Every positive number has two square roots. The number 0 has only one square root, 0.

If N is a positive number, then $\sqrt{N}$ indicates the positive square root of N. For example, $\sqrt{4} = 2$. The negative square root of 4 is $-\sqrt{4} = -2$.

If the area of a square is known, then square roots can be used to describe the length of a side of the square.

Getting Ready for Problem 2.2

- What is the side length of a square with an area of 2 square units?
- Is this length greater than 1? Is it greater than 2?
- Is 1.5 a good estimate for $\sqrt{2}$?
- Can you find a better estimate for $\sqrt{2}$?



Problem 2.2 Square Roots

In this problem, use your calculator only when the question directs you to.

A. **1.** Find the side lengths of squares with areas of 1, 9, 16, and 25 square units.

2. Find the values of $\sqrt{1}$, $\sqrt{9}$, $\sqrt{16}$, and $\sqrt{25}$.

B. **1.** What is the area of a square with a side length of 12 units? What is the area of a square with a side length of 2.5 units?

2. Find the missing numbers.

$\sqrt{\square} = 12$ $\sqrt{\square} = 2.5$

C. Refer to the square with an area of 2 square units you drew in Problem 2.1. The exact side length of this square is $\sqrt{2}$ units.

1. Estimate $\sqrt{2}$ by measuring a side of the square with a centimeter ruler.

2. Calculate the area of the square, using your measurement from part (1). Is the result exactly equal to 2?

3. Use the square root key on your calculator to estimate $\sqrt{2}$.

4. How does your ruler estimate compare to your calculator estimate?

D. **1.** Which two whole numbers is $\sqrt{5}$ between? Explain.

2. Which whole number is closer to $\sqrt{5}$? Explain.

3. Without using the square root key on your calculator, estimate the value of $\sqrt{5}$ to two decimal places.

E. Give the exact side length of each square you drew in Problem 2.1.

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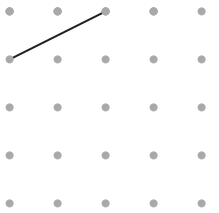
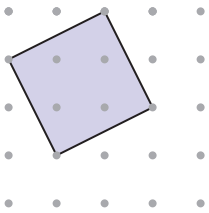
2.3 Using Squares to Find Lengths

You can use a square to find the length of a segment connecting dots on a grid. For example, to find the length of the segment on the left, draw a square with the segment as a side. The square has an area of 5 square units, so the segment has length $\sqrt{5}$ units.





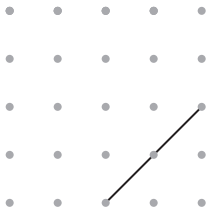
Problem 2.3 Using Squares to Find Lengths

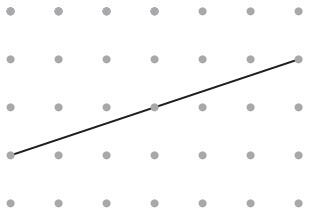
A. **1.** On 5 dot-by-5 dot grids, draw line segments with as many different lengths as possible by connecting dots. Label each segment with its length. Use the $\sqrt{\ }$ symbol to express lengths that are not whole numbers. (**Hint:** You will need to draw squares that extend beyond the 5-dot-by-5-dot grids.)

2. List the lengths in increasing order.

3. Estimate each non-whole number length to one decimal place.

B. Ella says the length of the segment at the left below is $\sqrt{8}$ units. Isabel says it is $2\sqrt{2}$ units. Are both students correct? Explain.





C. **1.** Question B gives two ways of expressing the exact length of a segment. Express the exact length of the segment at the right above in two ways.

2. Can you find a segment whose length cannot be expressed in two ways as in Question B?

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