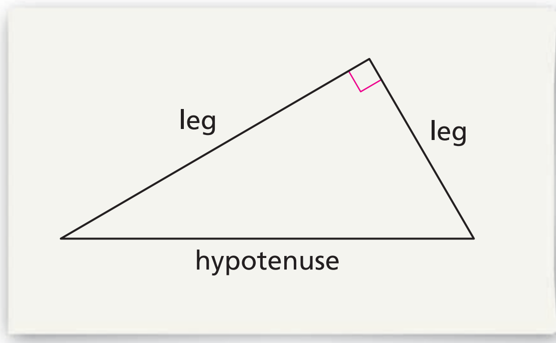
Investigation 3

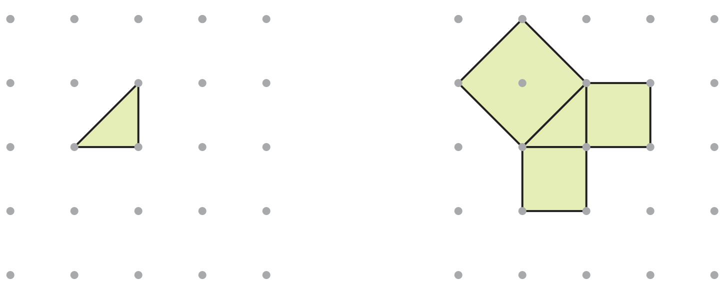
The Pythagorean Theorem

Recall that a right triangle is a triangle with a right, or 90°, angle. The longest side of a right triangle is the side opposite the right angle. We call this side the **hypotenuse** of the triangle. The other two sides are called the **legs.** The right angle of a right triangle is often marked with a square.



3.1 The Pythagorean Theorem

Each leg of the right triangle on the left below has a length of 1 unit. Suppose you draw squares on the hypotenuse and legs of the triangle, as shown on the right.



How are the areas of the three squares related?

In this problem, you will look for a relationship among the areas of squares drawn on the sides of right triangles.

Problem 3.1 The Pythagorean Theorem

A. Copy the table below. For each row of the table:

- Draw a right triangle with the given leg lengths on dot paper.
- Draw a square on each side of the triangle.
- Find the areas of the squares and record the results in the table.

Length of Leg 1 (units)	Length of Leg 2 (units)	Area of Square on Leg 1 (square units)	Area of Square on Leg 2 (square units)	Area of Square on Hypotenuse (square units)
1	1	1	1	2
1	2	■	■	■
2	2	■	■	■
1	3	■	■	■
2	3	■	■	■
3	3	■	■	■
3	4	■	■	■

B. Recall that a **conjecture** is your best guess about a mathematical relationship. It is usually a generalization about a pattern you think might be true, but that you do not yet know for sure is true.

For each triangle, look for a relationship among the areas of the three squares. Make a conjecture about the areas of squares drawn on the sides of any right triangle.

C. Draw a right triangle with side lengths that are different than those given in the table. Use your triangle to test your conjecture from Question B.

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3.2 A Proof of the Pythagorean Theorem

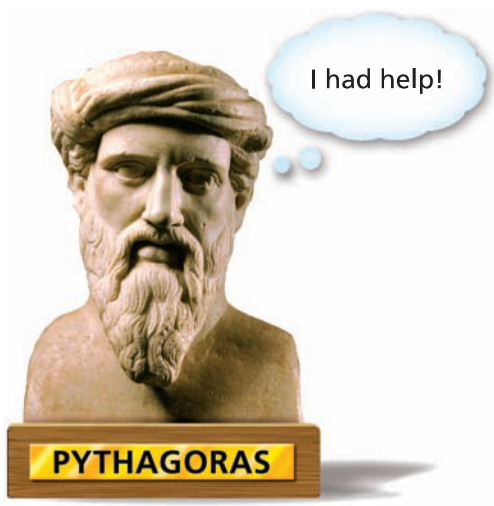
The pattern you discovered in Problem 3.1 is a famous theorem named after the Greek mathematician Pythagoras. A *theorem* is a general mathematical statement that has been proven true. The Pythagorean Theorem is one of the most famous theorems in mathematics.

Over 300 different proofs have been given for the Pythagorean Theorem. One of these proofs is based on the geometric argument you will explore in this problem.

Did You Know?

Pythagoras lived in the sixth century B.C. He had a devoted group of followers known as the Pythagoreans.

The Pythagoreans were a powerful group. Their power and influence became so strong that some people feared they threatened the local political structure, and they were forced to disband. However, many Pythagoreans continued to meet in secret and to teach Pythagoras's ideas.



Because they held Pythagoras in such high regard, the Pythagoreans gave him credit for all of their discoveries. Much of what we now attribute to Pythagoras, including the Pythagorean Theorem, may actually be the work of one or several of his followers.

Go Online PHSchool.com **For:** Information about Pythagoras
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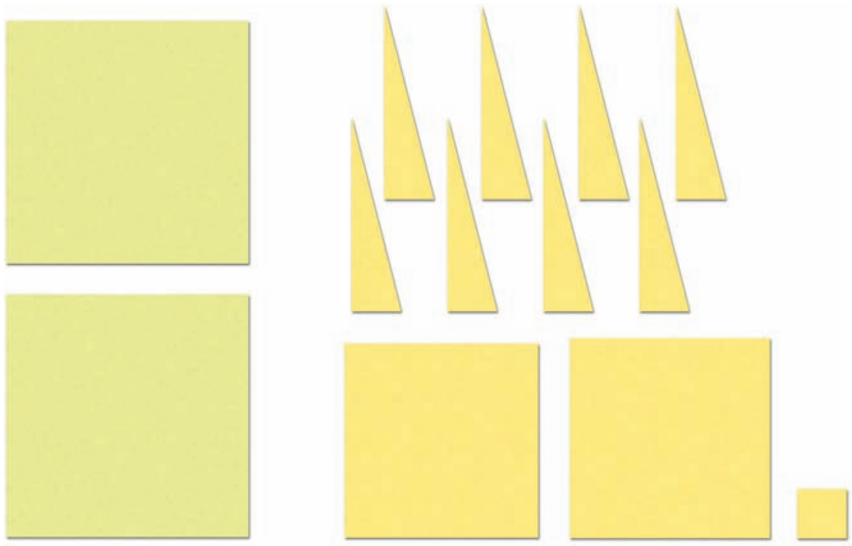
Problem 3.2 A Proof of the Pythagorean Theorem

Use the puzzles your teacher gives you.



Puzzle frames Puzzle pieces

A. Study a triangle piece and the three square pieces. How do the side lengths of the squares compare to the side lengths of the triangle?

B. **1.** Arrange the 11 puzzle pieces to fit exactly into the two puzzle frames. Use four triangles in each frame.

2. What conclusion can you draw about the relationship among the areas of the three squares?

3. What does the conclusion you reached in part (2) mean in terms of the side lengths of the triangles?

4. Compare your results with those of another group. Did that group come to the same conclusion your group did? Is this conclusion true for all right triangles? Explain.

C. Suppose a right triangle has legs of length 3 centimeters and 5 centimeters.

1. Use your conclusion from Question B to find the area of a square drawn on the hypotenuse of the triangle.

2. What is the length of the hypotenuse?

D. In this Problem and Problem 3.1, you explored the Pythagorean Theorem, a relationship among the side lengths of a right triangle. State this theorem as a rule for any right triangle with leg lengths a and b and hypotenuse length c.

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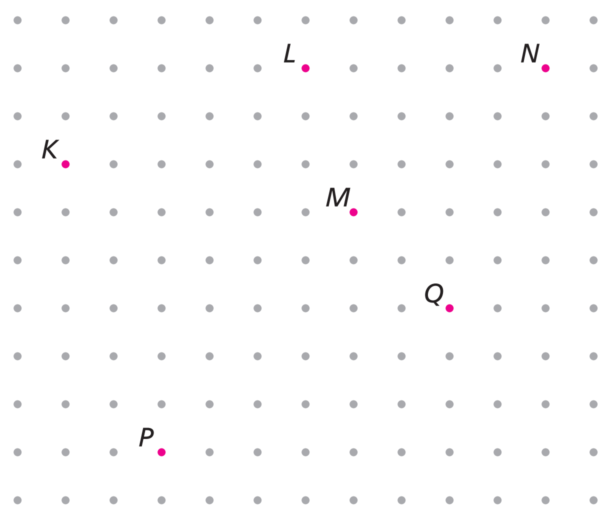
3.3 Finding Distances

In Investigation 2, you found the lengths of tilted segments by drawing squares and finding their areas. You can also find these lengths using the Pythagorean Theorem.

Problem 3.3 Finding Distances

In Questions A–D, refer to the grid below.



A. **1.** Copy the points above onto dot paper. Draw a right triangle with segment KL as its hypotenuse.

2. Find the lengths of the legs of the triangle.

3. Use the Pythagorean Theorem to find the length of segment KL.

B. Find the distance between points M and N by connecting them with a segment and using the method in Question A.

C. Find the distance between points P and Q.

D. Find two points that are $\sqrt{13}$ units apart. Label the points X and Y. Explain how you know the distance between the points is $\sqrt{13}$ units.

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3.4 Measuring the Egyptian Way

You will now explore these questions about the Pythagorean Theorem:

- Is any triangle whose side lengths a, b, and c, satisfy the relationship $a^2 + b^2 = c^2$ a right triangle?
- Suppose the side lengths of a triangle do *not* satisfy the relationship $a^2 + b^2 = c^2$. Does this mean the triangle is *not* a right triangle?

Getting Ready for Problem 3.4

In ancient Egypt, the Nile River overflowed every year, flooding the surrounding lands and destroying property boundaries. As a result, the Egyptians had to remeasure their land every year.

Because many plots of land were rectangular, the Egyptians needed a reliable way to mark right angles. They devised a clever method involving a rope with equally spaced knots that formed 12 equal intervals.

To understand the Egyptians' method, mark off 12 segments of the same length on a piece of rope or string. Tape the ends of the string together to form a closed loop. Form a right triangle with side lengths that are whole numbers of segments.

- What are the side lengths of the right triangle you formed?
- Do the side lengths satisfy the relationship $a^2 + b^2 = c^2$?
- How do you think the Egyptians used the knotted rope?

Problem 3.4 Lengths That Form a Right Triangle

A. Copy the table below. Each row gives three side lengths. Use string, straws, or polystrips to build a triangle with the given side lengths. Then, complete the second and third columns of the table.

Side Lengths (units)	Do the side lengths satisfy $a^2 + b^2 = c^2$?	Is the triangle a right triangle?
3, 4, 5		
5, 12, 13		
5, 6, 10		
6, 8, 10		
4, 4, 4		
1, 2, 2		

B. **1.** Make a conjecture about triangles whose side lengths satisfy the relationship $a^2 + b^2 = c^2$.

2. Make a conjecture about triangles whose side lengths do not satisfy the relationship $a^2 + b^2 = c^2$.

3. Check your conjecture with two other triangles. Explain why your conjecture will always be true.

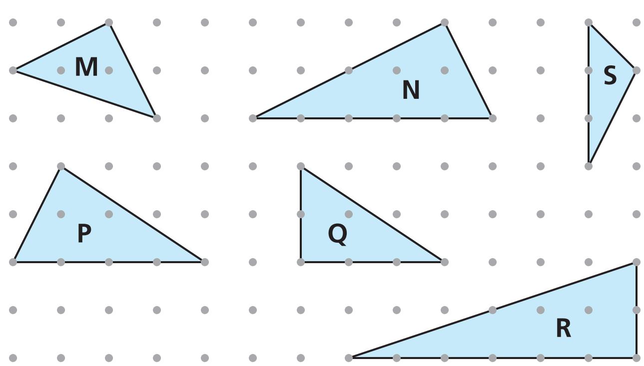
C. Determine whether the triangle with the given side lengths is a right triangle.

1. 12 units, 16 units, 20 units

2. 8 units, 15 units, 17 units

3. 12 units, 9 units, 16 units

D. Which of these triangles are right triangles? Explain.



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