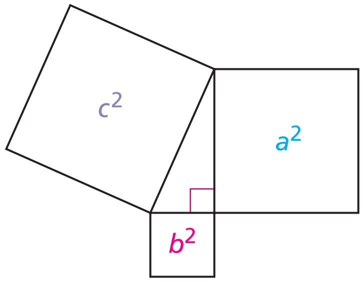
Investigation 4

Using the Pythagorean Theorem

In Investigation 3, you studied the Pythagorean Theorem, which states:

> The area of the square on the hypotenuse of a right triangle is equal to the sum of the areas of the squares on the legs.

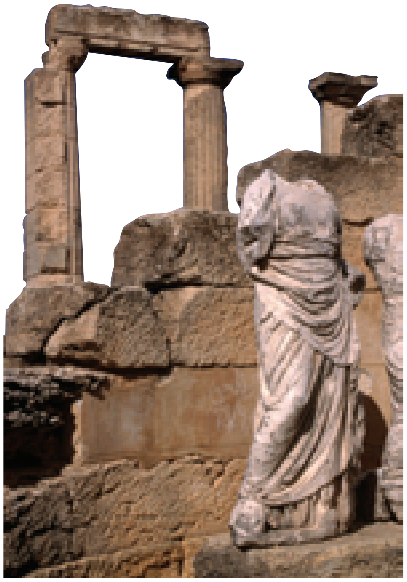


$a^2 + b^2 = c^2$

In this investigation, you will explore some interesting applications of the Pythagorean Theorem.

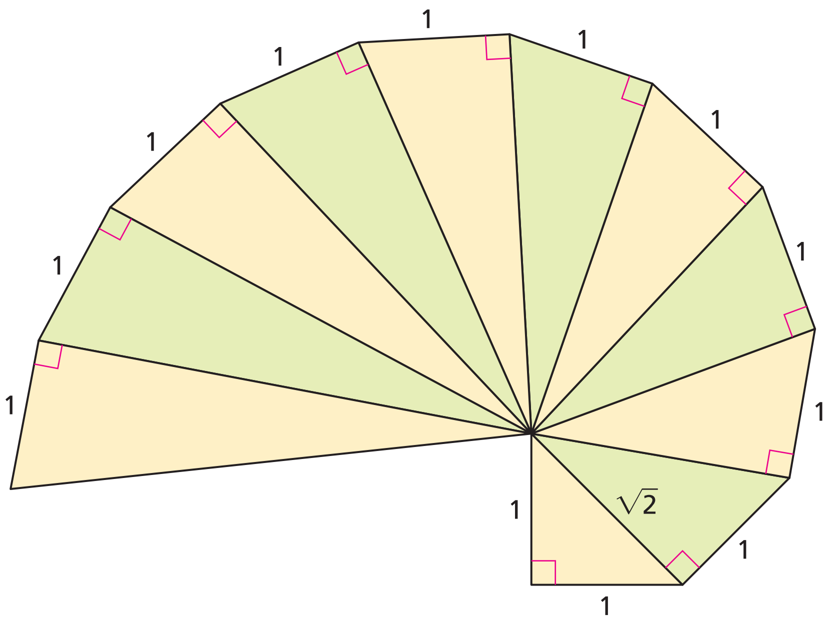
4.1 Analyzing The Wheel of Theodorus

The diagram on the next page is named for its creator, Theodorus of Cyrene (sy ree nee), a former Greek colony. Theodorus was a Pythagorean.



The Wheel of Theodorus begins with a triangle with legs 1 unit long and winds around counterclockwise. Each triangle is drawn using the hypotenuse of the previous triangle as one leg and a segment of length 1 unit as the other leg. To make the Wheel of Theodorus, you need only know how to draw right angles and segments 1 unit long.

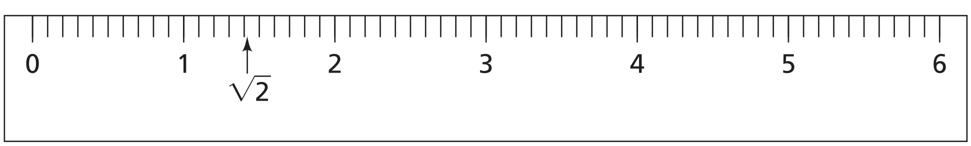
Wheel of Theodorus



Problem 4.1 Analyzing the Wheel of Theodorus

A. Use the Pythagorean Theorem to find the length of each hypotenuse in the Wheel of Theodorus. On a copy of the wheel, label each hypotenuse with its length. Use the $\sqrt{}$ symbol to express lengths that are not whole numbers.

B. Use a cut-out copy of the ruler below to measure each hypotenuse on the wheel. Label the place on the ruler that represents the length of each hypotenuse. For example, the first hypotenuse length would be marked like this:



C. For each hypotenuse length that is not a whole number:

1. Give the two consecutive whole numbers the length is between. For example, $\sqrt{2}$ is between 1 and 2.
2. Use your ruler to find two decimal numbers (to the tenths place) the length is between. For example $\sqrt{2}$ is between 1.4 and 1.5.
3. Use your calculator to estimate the value of each length and compare the result to the approximations you found in part (2).

D. Odakota uses his calculator to find $\sqrt{3}$. He gets 1.732050808. Geeta says this must be wrong because when she multiplies 1.732050808 by 1.732050808, she gets 3.000000001. Why do these students disagree?

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Did You Know?

Some decimals, such as 0.5 and 0.3125, *terminate*. They have a limited number of digits. Other decimals, such as 0.3333 . . . and 0.181818 . . . , have a repeating pattern of digits that never ends.

Terminating or repeating decimals are called **rational numbers** because they can be expressed as *ratios* of integers.

$$0.5 = \frac{1}{2} \qquad 0.3125 = \frac{5}{16} \qquad 0.3333\ldots = \frac{1}{3} \qquad 0.181818\ldots = \frac{2}{11}.$$

Some decimals neither terminate nor repeat. The decimal representation of the number π starts with the digits 3.14159265 . . . and goes forever without any repeating sequence of digits. Numbers with non-terminating and non-repeating decimal representations are called **irrational numbers.** They cannot be expressed as ratios of integers.

The number $\sqrt{2}$ is an irrational number. You had trouble finding an exact terminating or repeating decimal representation for $\sqrt{2}$ because such a representation does not exist. Other irrational numbers are $\sqrt{3}$, $\sqrt{5}$, and $\sqrt{11}$. In fact, $\sqrt{n}$ is an irrational number for any value of n that is not a square number.

The set of irrational and rational numbers is called the set of **real numbers.** An amazing fact about irrational numbers is that there is an infinite number of them between any two fractions!

4.2 Stopping Sneaky Sally

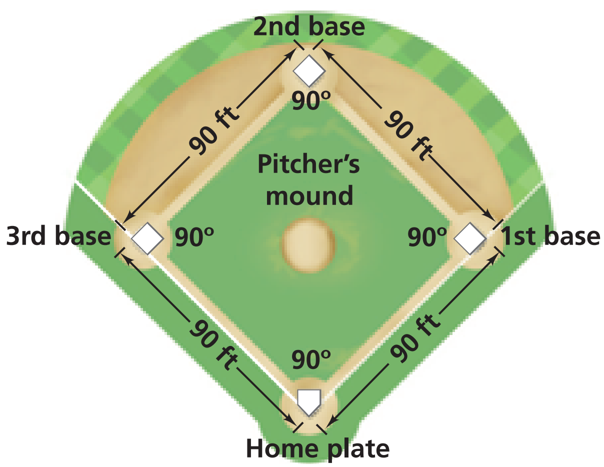
You can use the Pythagorean Theorem to solve problems in which you need to find the length of a side of a right triangle.

Problem 4.2 Finding Unknown Side Lengths

Horace Hanson is the catcher for the Humboldt Bees baseball team. Sneaky Sally Smith, the star of the Canfield Cats, is on first base. Sally is known for stealing bases, so Horace is keeping an eye on her.

The pitcher throws a fastball, and the batter swings and misses. Horace catches the pitch and, out of the corner of his eye, he sees Sally take off for second base.

Use the diagram to answer Questions A and B.



A. How far must Horace throw the baseball to get Sally out at second base? Explain.

B. The shortstop is standing on the baseline, halfway between second base and third base. How far is the shortstop from Horace?

C. The pitcher's mound is 60 feet 6 inches from home plate. Use this information and your answer to Question A to find the distance from the pitcher's mound to each base.

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Did You Know?

Although most people consider baseball an American invention, a similar game, called *rounders*, was played in England as early as the 1600s. Like baseball, rounders involved hitting a ball and running around bases. However, in rounders, the fielders actually threw the ball at the base runners. If a ball hit a runner while he was off base, he was out.

Alexander Cartwright was a founding member of the Knickerbockers Base Ball Club of New York City, baseball's first organized club. Cartwright played a key role in writing the first set of formal rules for baseball in 1845.

According to Cartwright's rules, a batter was out if a fielder caught the ball either on the fly or on the first bounce. Today, balls caught on the first bounce are not outs. Cartwright's rules also stated that the first team to have 21 runs at the end of an inning was the winner. Today, the team with the highest score after nine innings wins the game.

Go Online PHSchool.com **For:** Information about Alexander Cartwright
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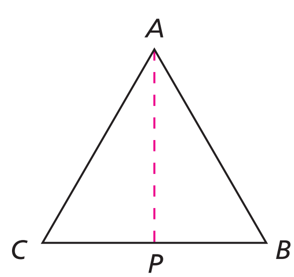
4.3 Analyzing Triangles

All equilateral triangles have reflection symmetries. This property and the Pythagorean Theorem can be used to investigate some interesting properties of other equilateral triangles.

Getting Ready for Problem 4.3

Triangle ABC is an equilateral triangle.

- What is true about the angle measures in an equilateral triangle?
- What is true about the side lengths of an equilateral triangle?



Line *AP* is a reflection line for triangle *ABC*.

- What can you say about the measures of the following angles? Explain.

Angle *CAP*	Angle *BAP*
Angle *CPA*	Angle *BPA*

- What can you say about line segments *CP* and *PB*? Explain.
- What can you say about triangles *ACP* and *ABP*?

Problem 4.3 Analyzing Triangles

A. Copy triangle *ABC* on the facing page. If the lengths of the sides of this equilateral triangle are 4 units, label the following measures:

1. angle *CAP* **2.** angle *BAP*

3. angle *CPA* **4.** angle *BPA*

5. length of *CP* **6.** length of *PB*

7. length of *AP*

B. Suppose the lengths of the sides of *ABC* triangles are *s* units. Find the measures of the following:

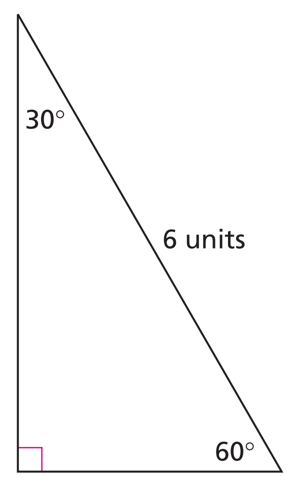
1. angle *CAP* **2.** angle *BAP*

3. angle *CPA* **4.** angle *BPA*

5. length of *CP* **6.** length of *PB*

7. length of *AP*

C. A right triangle with a 60° angle is called a 30-60-90 triangle. This 30-60-90 triangle has a hypotenuse of length 6 units.



1. What are the lengths of the other two sides? Explain how you found your answers.

2. What relationships among the side lengths do you observe for this 30-60-90 triangle? Is this relationship true for all 30-60-90 triangles? Explain.

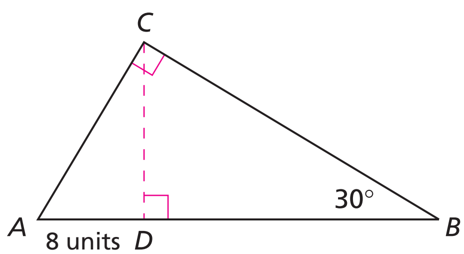
ACE **Homework starts on page 53.**

4.4 Finding the Perimeter

In this problem, you will apply many of the strategies you have developed in this unit, especially what you found in Problem 4.3.

Problem 4.4 Finding the Perimeter

Use the diagram for Questions A–C. Explain your work.



A. Find the perimeter of triangle *ABC*.

B. Find the area of triangle *ABC*.

C. Find the areas of triangle *ACD* and triangle *BCD*.

ACE **Homework starts on page 53.**

Did You Know?

In the movie *The Wizard of Oz,* the scarecrow celebrates his new brain by reciting the following:

"The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side."

Now you know what the scarecrow meant to say, even though his still imperfect brain got it wrong!